DOCUMENT RESUME

ED 164 306 SE 025 461

AUTHOR	Beatty, Leslie; And Others
TITLE	Mathematics for the Elementary School, Book 3, Teacher's Commentary, Part II, Unit No. 59. Revised Edition.
INSTITUTION	Stanford Univ., Calif. School Mathematics Study Group.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	65
NOTE	352p.; For related document, see SE 025 460
EDRS PRICE	MF-$0.83 HC-$19.41 Plus Postage.
DESCRIPTORS	Curriculum; Elementary Education; *Elementary School Mathematics; *Geometry; *Instructional Materials; Mathematics Education; *Number Concepts; *Textbooks
IDENTIFIERS	*School Mathematics Study Group

ABSTRACT

This is part two of a two-part SMSG mathematics text for elementary school students. One of the goals of the text is the development of mathematical ideas via appropriate experiences with things from the physical world and the immediate environment. The text materials provide an introduction to the study of mathematics in which growth is from the concrete to the abstract, from the specific to the general. The authors emphasize exploration and progressive refinement of ideas associated with both number and space. Chapter topics include: (1) addition and subtraction - shorter forms of computation; (2) length and area; (3) multiplication, quotients, and division; (4) rational numbers; and (5) division. (MP)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U S DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

SMSG

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) AND USERS OF THE ERIC SYSTEM."

ED164306

SE 025 461

Mathematics for the Elementary School

Book 3

Teacher's Commentary, Part II

REVISED EDITION

Prepared under the supervision of the
Panel on Elementary School Mathematics
of the School Mathematics Study Group:

Leslie Beatty	Chula Vista City School District, Chula Vista, California
E. Glenadine Gibb	State College of Iowa
William T. Guy	University of Texas
Stanley B. Jackson	University of Maryland
Irene Sauble	Detroit Public Schools
Marshall H. Stone	University of Chicago
J. Fred Weaver	Boston University
Raymond L. Wilder	University of Michigan

New Haven and London, Yale University Press, 1965

Copyright © 1965 by The Board of Trustees
of the Leland Stanford Junior University.
Printed in the United States of America.

All rights reserved. This book may not
be reproduced in whole or in part, in
any form, without written permission from
the publishers.

Financial support for the School Mathematics
Study Group has been provided by the
National Science Foundation.

TABLE OF CONTENTS

Chapter

V ADDITION AND SUBTRACTION: SHORTER FORMS OF COMPUTATION . . . 403
V- 1. Adding and Subtracting Multiples of Ten and One Hundred . . . 405
V- 2. A "Short Form", Renaming Once . . . 422
V- 3. Finding The Sum of Three Numbers . . . 438
V- 4. Adding, with Renaming More than Once . . . 451
V- 5. Subtracting with Renaming More than Once . . . 466

VI LENGTH AND AREA . . . 513
VI- 1. Length of a Curve . . . 518
VI- 2. Perimeters of Polygons . . . 526
VI- 3. Using Different Units . . . 537
VI- 4. Using Fractional Units . . . 545
VI- 5. Introduction to Area . . . 552
VI- 6. Doubling Edges of Rectangles . . . 565
VI- 7. Congruence and Area . . . 575
VI- 8. Causs Sums and Euler Routes . . . 597

VII MULTIPLICATION, QUOTIENTS, AND DIVISION . . . 607
VII- 1. Multiplying with 10 . . . 613
VII- 2. Using Multiplication to Solve Problems . . . 619
VII- 3. Division . . . 625
VII- 4. Quotients . . . 631
VII- 5. Relating Division to Multiplication . . . 638
VII- 6. The Associative Property of Multiplication . . . 645
VII- 7. Multiplying and Dividing 10 and Multiples of 10 . . . 652
VII- 8. The Distributive Property . . . 660
VII- 9. Solving Problems . . . 667
VII- 10. Factors and Clock Wheels (Optional) . . . 676

VIII RATIONAL NUMBERS . . . 683
VIII- 1. Rational Numbers Associated with Parts of Regions . . . 685
VIII- 2. Rational Numbers Associated with Subsets of a Set . . . 690
VIII- 3. Identifying a Subset as a Part of a Set . . . 693
VIII- 4. Finding a Part of a Set . . . 698
VIII- 5. Rational Numbers Describe Points of the Number Line . . . 703
VIII- 6. Order Among Rational Numbers . . . 706

IX DIVISION . . . 713
IX- 1. Division Concepts . . . 716
IX- 2. Finding Quotients . . . 725
IX- 3. Finding the Number that Names Parts of a Set . . . 730
IX- 4. A Technique for Finding Quotients . . . 738
IX- 5. Solving Problems . . . 744

Chapter V

ADDITION AND SUBTRACTION: SHORTER FORMS OF COMPUTATION

Background

Regrouping and renaming

These terms are used in a restricted sense in this chapter. Regrouping applies only to sets and their members. It indicates essentially a change of grouping. For example, we may have one set which has twenty members and another which has fourteen members. We may choose to regard these as two sets with ten members each and one set with fourteen members. Or, if it suits our convenience, we may regard them as three sets with ten members each and one set with four members.

Renaming applies to numbers only. It indicates a change from one representation in the decimal system of numeration to another representation in the same system.

For example, if we have the number three hundred fifty-four, it may be represented as

$$300 + 50 + 4$$

or we may choose to regard it as

$$200 + 140 + 14.$$

Using shorter forms

As children develop techniques for computing sums and differences, they become skilled in selecting appropriate numerals for convenience in computing. Also, they become more skillful in sensing short-cuts they can use to compute sums or differences of two numbers.

For example, at this time some children may be learning to compute the sum of 246 and 147 by using the form

$$\begin{array}{r} 246 \\ \underline{147} \\ 300 \\ 80 \\ \underline{13} \\ 393 \end{array} \qquad \text{instead of} \qquad \begin{array}{r} 200 + 40 + 6 \\ \underline{100 + 40 + 7} \\ 300 + 80 + 13 \end{array}$$

Others may be learning to use

$$\begin{array}{r} 246 \\ \underline{147} \\ 393 \end{array} \qquad \text{instead of} \qquad \begin{array}{r} 246 \\ \underline{147} \\ 300 \\ 80 \\ \underline{13} \\ 393 \end{array}$$

Other children may be developing even shorter and more individualized methods.

You can expect skill in finding differences to be developed in a similar way.

Skill is an individual matter. You should not impose a short form upon a child, or expect all children to be using the same computational procedures. Yet each child should be encouraged and urged to improve upon those techniques and skills that he has already achieved.

V-1. Adding and subtracting multiples of ten and one hundred

Objective: To develop skill in adding and subtracting multiples of ten and one hundred.

Vocabulary: Decimal numeral.

Suggested Procedure:

Ask children to count by tens to 300 or more. Have them take turns, with each child saying only three or four numbers. (Bill: "Ten, twenty, thirty"; Sue: "Forty, fifty, sixty"; Ed: "Seventy, eighty, ninety"; etc.)

Repeat, beginning with ten but stop the counting at 100.

How many tens is 100 ?	(10.)
How many tens is 110 ?	(11.)
How many tens is 200 ?	(20.)
If we think of 365 as tens and ones, how many tens are there?	(36.)

This may need considerable emphasis if children have been thinking of 365 only as 300 + 60 + 5, or as 3 hundreds, 6 tens, and 5 ones. If they have difficulty, remind them that each hundred is the same as 10 tens, so 3 hundreds is 30 tens plus 6 tens or 36 tens. You may wish to have the class count by tens while you write the numerals on the chalkboard, and then to the side of each put the numeral for the number of tens:

10 = 1 ten
20 = 2 tens

.
.
.

100 = 10 tens
110 = 11 tens

.
.
.

200 = 20 tens
210 = 21 tens

etc.

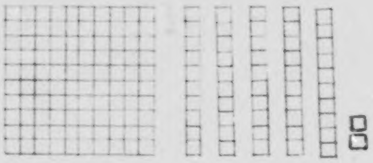
Explain that the usual way of expressing 21 tens (or 2 hundreds, 1 ten, and 0 ones) is the decimal numeral 210. The word "decimal" comes from the Latin word decem, meaning ten. In our place value system, the position of a digit in the usual way of expressing a number shows whether the digit refers to ones, tens, or tens of tens, etc. A numeral which makes use of this place value system is called a decimal numeral.

Pupil's book, page 235: Discuss this page with the children.

Thinking about Numbers as Tens and Ones

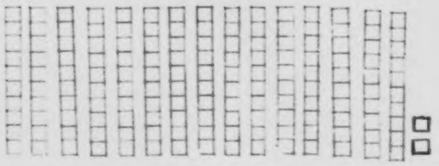
Mary had 152 stamps. She tried arranging them in different ways.

She thought of 152 as:



She wrote 1 hundred 5 tens 2 ones.

Then she thought of 152 as:



She wrote 15 tens 2 ones.

Think of numbers as tens and ones:

Remember: 1 hundred = 10 tens.

186 = 18 tens 6 ones

200 = 20 tens 0 ones

345 = 34 tens 5 ones

727 = 72 tens 7 ones

504 = 50 tens 4 ones

317 = 31 tens 7 ones

499 = 49 tens 9 ones

782 = 78 tens 2 ones

420 = 42 tens 0 ones

700 = 70 tens 0 ones

129 = 12 tens 9 ones

870 = 87 tens 0 ones

603 = 60 tens 3 ones

890 = 89 tens 0 ones

- Review the idea of adding and subtracting multiples of ten.

  How many tens is the number 210? (21.)
  Add 3 tens to 21 tens. What is their sum? (24 tens.)
  What is the decimal numeral for that number? (240.)
  How many tens and ones is 435? (43 tens and 5 ones.)

  Do as much of the following orally as possible.

  Ask children to start with 10 and add 30 each time, taking turns, one at a time. (10, 40, 70, etc.)

  Start with 430 and count backward by tens.

  Start with 210 and subtract 20 each time, to the smallest whole number obtainable.

  Start with 26 and add 10 each time to 200 or beyond.

  Start with 195 and subtract 10 each time, to the smallest whole number obtainable.

- Send several children to the chalkboard with instructions to start with a given number and add or subtract a multiple of ten a certain number of times (e.g., start with 691 and subtract 30 three times; start with 329 and add 20 four times). The pupils who are at their desks may do the same exercise using paper and pencil. If children need to use the addition algorithm, do not discourage them, but it is hoped that many will be able to do the work mentally and write only the results.

- Have the children count by hundreds starting with 100, with 142, etc. Have them count backward by hundreds, starting with 798, to the smallest whole number obtainable. Send several children to the chalkboard to do similar work (e.g., start with 114 and add 200 four times; start with 982, subtract 200 four times). Those pupils who remain at their seats may do the exercise using paper and pencil.

Be sure the children realize that when we count by 10 as in 10, 20, 30, 40, etc., these numbers are known as multiples of 10. The same emphasis should be made for multiples of 100.

Pupil's book, pages 236-243:

These pages provide practice in adding and subtracting multiples of ten and multiples of a hundred. It should be clear to the children that when a multiple of ten is added to a number (no renaming required) the sum will have the same digits as the first addend except in the tens place. A similar observation should be made when a multiple of a hundred is added, that is, the digits in the sum will be the same as the digits in the first addend except in the hundreds place.

● Finding Patterns

Generalization depends upon the ability to recognize similarities in varied situations and experiences. Children are not likely to develop this ability if they are not made aware that similarities exist.

In this lesson children are given the opportunity to find the patterns in number sequences. It is probably necessary to help children develop a strategy for attacking the problem.

Write on the chalkboard:

4, 8, 12, 16, __, __, __

Tell the children that the sequence has a pattern. Ask if anyone knows what it is. Once 20, 24 and 28 are established as the numbers which are in the sequence, ask several children to explain how they arrived at their answers. A possible strategy is:

Adding a Multiple of Ten

$\begin{array}{r}45\\+10\\\hline 55\end{array}$	$\begin{array}{r}45\\+20\\\hline 65\end{array}$	$\begin{array}{r}45\\+30\\\hline 75\end{array}$	$\begin{array}{r}45\\+40\\\hline 85\end{array}$	$\begin{array}{r}45\\+50\\\hline 95\end{array}$

23 + 30 = 53

58 + 10 = 68

19 + 40 = 59

47 + 50 = 97

66 + 30 = 96

25 + 70 = 95

55 + 30 = 85

26 + 60 = 86

65 + 30 = 95

55 + 30 = 85

$\begin{array}{r}63\\+30\\\hline 93\end{array}$	$\begin{array}{r}26\\+20\\\hline 46\end{array}$	$\begin{array}{r}32\\+50\\\hline 82\end{array}$
$\begin{array}{r}35\\+20\\\hline 55\end{array}$	$\begin{array}{r}35\\+40\\\hline 75\end{array}$	$\begin{array}{r}22\\+70\\\hline 92\end{array}$
$\begin{array}{r}31\\+40\\\hline 71\end{array}$	$\begin{array}{r}33\\+40\\\hline 73\end{array}$	$\begin{array}{r}78\\+20\\\hline 98\end{array}$
$\begin{array}{r}47\\+30\\\hline 77\end{array}$	$\begin{array}{r}52\\+40\\\hline 92\end{array}$	$\begin{array}{r}65\\+30\\\hline 95\end{array}$
$\begin{array}{r}47\\+20\\\hline 67\end{array}$	$\begin{array}{r}85\\+10\\\hline 95\end{array}$	$\begin{array}{r}43\\+50\\\hline 93\end{array}$
$\begin{array}{r}68\\+30\\\hline 98\end{array}$	$\begin{array}{r}74\\+20\\\hline 94\end{array}$	$\begin{array}{r}62\\+30\\\hline 92\end{array}$
$\begin{array}{r}46\\+40\\\hline 86\end{array}$	$\begin{array}{r}24\\+70\\\hline 94\end{array}$	$\begin{array}{r}57\\+30\\\hline 87\end{array}$

Subtracting a Multiple of Ten

95	95	95	95	95	95
-10	-20	-30	-40	-50	-60
85	75	65	55	45	35

88	77	49
-30	-10	-30
58	67	19
36	85	63
-20	-40	-10
16	45	53
47	95	59
-20	-20	-50
27	75	9
56	74	67
-10	-70	-20
46	4	47
83	97	53
-60	-40	-40
23	57	13
98	86	74
-70	-60	-50
18	26	24
62	81	58
-50	-60	-30
12	21	28

53 - 30 = 23

48 - 10 = 38

59 - 40 = 19

46 - 30 = 16

57 - 10 = 47

40 - 20 = 20

85 - 10 = 75

96 - 20 = 76

75 - 10 = 65

22 - 20 = 2

Adding a Multiple of Ten

$\begin{array}{r}237\\+10\\\hline 247\end{array}$ $\begin{array}{r}237\\+20\\\hline 257\end{array}$ $\begin{array}{r}237\\+30\\\hline 267\end{array}$ $\begin{array}{r}237\\+40\\\hline 277\end{array}$

1. Add 10 to each number. Write the sum.

237	556	738	674
247	566	748	684

2. Add 20 to each number. Write the sum.

656	324	148	437
676	344	168	457

3. Add 40 to each number. Write the sum.

320	730	130	450
360	770	170	490

4. **Adding 10**

137 + 10 = 147
103 + 10 = 113
143 + 10 = 153

5. **Adding 20**

137 + 20 = 157
103 + 20 = 123
735 + 20 = 755

Adding Ones, Tens, or Hundreds

$$\begin{array}{r} 147 \\ +\ \ 2 \\ \hline 149 \end{array} \qquad \begin{array}{r} 147 \\ +\ 20 \\ \hline 167 \end{array} \qquad \begin{array}{r} 147 \\ +200 \\ \hline 347 \end{array}$$

Add the number named at the top of each column to the number named in the left-hand column. Write each sum.

	Add 6	Add 60	Add 600
320	326	380	920
111	117	171	711
332	338	392	932
432	438	492	1032

	Add 2	Add 20	Add 200
147	149	167	347
403	405	423	603
637	639	657	837
713	715	733	913

Subtracting a Multiple of Ten or a Hundred

523 - 10 513	523 -200 323

830 -20 810	690 -80 610
564 - 30 534	759 -40 719
738 - 30 708	572 - 40 532
157 - 30 127	286 - 40 246
482 - 50 432	389 - 300 89
691 - 300 391	273 -100 173
965 - 400	376 - 70

214 - 10 = 204

520 - 200 = 320

820 - 300 = 520

754 - 30 = 724

938 - 20 = 918

630 - 400 = 230

750 - 200 = 550

310 - 100 = 210

270 - 40 = 230

590 - 200 = 390

Adding and Subtracting Ones, Tens, Hundreds, and Thousands

Complete each equation.

3 + 4 = 7
30 + 40 = 70
300 + 400 = 700
3000 + 4000 = 7000

2 + 6 = 8
20 + 60 = 80
200 + 600 = 800
2000 + 6000 = 8000

4 + 5 = 9
40 + 50 = 90
400 + 500 = 900
4000 + 5000 = 9000

3 + 5 = 8
30 + 50 = 80
300 + 500 = 800
3000 + 5000 = 8000

9 - 5 = 4
90 - 50 = 40
900 - 500 = 400
9000 - 5000 = 4000

7 - 3 = 4
70 - 30 = 40
700 - 300 = 400
7000 - 3000 = 4000

8 - 3 = 5
80 - 30 = 50
800 - 300 = 500
8000 - 3000 = 5000

8 - 6 = 2
80 - 60 = 20
800 - 600 = 200
8000 - 6000 = 2000

Differences of One, Ten, or One Hundred.

Show the difference between the numbers:

36 and 46	100	[✓ 10]	1
28 and 29	100	10	[✓ 1]
7 and 17	100	[✓ 10]	1
92 and 82	100	[✓ 10]	1
47 and 57	100	[✓ 10]	1
145 and 135	100	[✓ 10]	1
875 and 775	[✓ 100]	10	1
987 and 986	100	10	[✓ 1]
776 and 766	100	[✓ 10]	1
58 and 68	100	[✓ 10]	1
340 and 440	[✓ 100]	10	1
209 and 210	100	10	[✓ 1]
487 and 477	100	[✓ 10]	1
509 and 609	[✓ 100]	10	1
301 and 311	100	[✓ 10]	1

Adding and Subtracting Multiples of Ten and a Hundred

Use either > (greater than), or < (less than) in each box to make the sentence true.

$40 + 50 \boxed{<} 100$

$159 + 40 \boxed{<} 200$

$33 + 50 \boxed{<} 100$

$90 + 97 \boxed{<} 200$

$18 + 30 \boxed{<} 50$

$556 + 400 \boxed{<} 1000$

$25 + 70 \boxed{<} 100$

$299 + 200 \boxed{>} 400$

$421 + 80 \boxed{>} 500$

$600 + 500 \boxed{>} 1000$

$556 + 400 \boxed{>} 900$

$900 - 700 \boxed{>} 100$

$85 - 70 \boxed{<} 20$

$59 - 40 \boxed{>} 10$

$46 - 20 \boxed{>} 20$

$78 - 30 \boxed{<} 50$

$156 - 40 \boxed{<} 100$

$790 - 80 \boxed{>} 700$

$44 - 30 \boxed{>} 10$

$59 - 40 \boxed{<} 20$

$665 - 60 \boxed{>} 600$

$432 - 50 \boxed{<} 400$

1) Look at the sequence. Are the numbers getting larger or smaller? (Larger.)

2) Since the numbers are getting larger, consider addition as a possibility. (Later in the school year multiplication would also be a possibility.)

3) Think about the first two numbers. In this case, what number added to 4 gives 8? (4.)

4) Try adding 4 to each number.

5) It works. The pattern is: Add 4.

Continue with some of the sequences given below:

a) 80, 78, 76, 74, __, __, __, __

Here the strategy must be changed.

1) The numbers are getting smaller. Try subtraction.

2) Thinking about the first two numbers suggests subtracting 2.

3) A check of the other numbers shows this is the pattern.

b) 24, 22, 25, 23, 26, __, __, __

Another change of strategy:

1) Numbers get smaller and then larger. This suggests subtraction, then addition.

2) Thinking about the first two numbers suggests subtracting 2.

3) Thinking about the second and third numbers suggests adding 3.

4) A check of the other numbers shows that subtracting 2 and then adding 3 is the pattern.

c) 100, 90, 80, 70, __, __, __ (subtract 10)

d) 5, 11, 17, 23, __, __, __ (add 6)

e) 7, 9, 8, 10, 9, 11, 10, __, __ (add 2, subtract 1)

f) 80, 75, 77, 72, 74, __, __, __, (subtract 5, add 2)

Pupil's book, pages 244 and 245:

These pages may be worked together or used for independent study depending on the ability of the class.

Patterns

Find the pattern.

1) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
What is the pattern? Add 2

2) 25, 22, 19, 16, 13, 10, 7, 4, 1
What is the pattern? Subtract 3

3) 70, 63, 56, 49, 42, 35, 28, 21, 14, 7, 0
What is the pattern? Subtract 7

4) 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
What is the pattern? Add 9

5) 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17
What is the pattern? Add 1, add 2, add 1, add 2...

6) 2, 9, 7, 14, 12, 19, 17, 24, 22, 19, 27
What is the pattern? Add 7, subtract 2

7) 10, 8, 13, 11, 16, 14, 19, 17, 22, 10, 25
What is the pattern? Subtract 2, add 5

8) 79, 83, 76, 80, 63, 67, 60, 64, 57
What is the pattern? Add 4, subtract 7

More about Patterns

Each list of numbers below has a pattern.

Can you tell what it is?

Fill in the blanks using this pattern.

(a) 210, 230, 250, 270, 290, 310
What is the pattern? Add 20.

(b) 67, 57, 47, 37, 27, 17
What is the pattern? Subtract 10

(c) 606, 636, 666, 696, 726
What is the pattern? Add 30.

(d) 900, 850, 800, 750, 700, 650
What is the pattern? Subtract 50.

(e) 253, 353, 453, 553, 653
What is the pattern? Add 100.

(f) 782, 762, 742, 722, 702
What is the pattern? Subtract 20.

(g) 347, 447, 547, 647, 747
What is the pattern? Add 100.

(h) 993, 963, 933, 903, 873
What is the pattern? Subtract 30.

V-2. A "short form", renaming once

Objectives: To develop use of a "short form" for adding and subtracting, including simple renaming situations.

To use "undoing" as a check for addition and subtraction examples.

Vocabulary: (No new words.)

Suggested Procedure:

Although children who need to use an expanded form for addition should not be prohibited from continuing to write (and think) 135 = 100 + 30 + 5, it is hoped that most of the class will acquire the habit of thinking of 135 as 1 hundred, 3 tens, and 5 ones, or, when it is helpful, as 13 tens and 5 ones, and will dispense with the expanded notation.

Addition

Present the following:

> Mary baked 84 cookies for a cookie sale. Her friend, Jean, baked 60 cookies. How many cookies did the two girls have to sell?

When children have formulated the equation, $84 + 60 = n$, ask how many tens and ones there are in 84 and how many tens and ones there are in 60.

> So to find the sum of 84 and 60, we add the tens, $8 + 6 = 14$, and the ones, $4 + 0 = 4$, and find that we have 14 tens and 4 ones or 144. We write this:

$$\begin{array}{r} 84 \\ +\ 60 \\ \hline 144 \end{array}$$

We think 8 tens + 6 tens = 14 tens,
and 4 ones + 0 ones = 4 ones;
14 tens and 4 ones are 144.

But suppose that Jean had broken two of the cookies and so had only 58. Then our equation would have been 84 + 58 = n. If we add the tens first, then the ones, we have to do a further addition to rename the ones:

$$\begin{array}{r} 84 \\ +\ 58 \\ \hline 130 \\ 12 \\ \hline 142 \end{array}$$

If instead, we begin with the ones, we can do the same work as before,

$$\begin{array}{r} 84 \\ +\ 58 \\ \hline 12 \\ 130 \\ \hline 142 \end{array}$$

or use a shorter form such as

$$\begin{array}{r} 84 \\ +\ 58 \\ \hline 142 \end{array}$$

Have children look at several examples written in the vertical form to determine whether it would be shorter to add the ones first or whether it does not make any difference.

- When many numbers less than 100 have been used, put the following on the chalkboard:

$$\begin{array}{r} 321 \\ +\ 546 \\ \hline 800 \\ 60 \\ 7 \\ \hline 867 \end{array} \qquad\qquad \begin{array}{r} 321 \\ +\ 546 \\ \hline 7 \\ 60 \\ 800 \\ \hline 867 \end{array} \qquad\qquad \begin{array}{r} 321 \\ +\ 546 \\ \hline 867 \end{array}$$

and show that the result is the same whether one adds the hundreds first or the ones first. Then write:

$$\begin{array}{r} 514 \\ +\ 277 \\ \hline \end{array}$$

Show the notation in the first two forms above, and ask children to tell how they might use a short form for such an example. Bring out the fact that it is helpful to begin with the ones because the sum of the ones is greater than 9. Show several examples of this kind, and ask whether it makes a difference whether you start with ones or with hundreds. Then write:

$$\begin{array}{r} 368 \\ +\ 441 \\ \hline \end{array}$$

Ask whether it would be more helpful to start with ones. Show that the sum of the tens will be greater than 9, so that there will have to be renaming.

Write several examples using numbers between 100 and 1000 and have children decide if renaming will be necessary.

● Subtraction.

To show that looking ahead in subtraction is useful in renaming, write

$$\begin{array}{r} 78 \\ -\ \ 8 \\ \hline \end{array}$$

and ask whether it is possible to decide whether to think of 78 as 7 tens and 8 ones or as 6 tens and 18 ones. Bring out the fact that knowing the tens digit for the number to be subtracted does not help in deciding. Children need to know the ones digit to determine whether to rename. If the number of ones is not greater than 8 (in this example), no renaming will be necessary. Give several examples like the following:

$$\begin{array}{r} 64 \\ \underline{-\ 33} \end{array} \qquad \begin{array}{r} 71 \\ \underline{-\ 59} \end{array} \qquad \begin{array}{r} 92 \\ \underline{-\ 62} \end{array} \qquad \begin{array}{r} 26 \\ \underline{-\ 19} \end{array}$$

Discuss whether or not renaming is necessary in each. Then have the children look at the example 135 - 42 = ___ . Express in vertical form and ask if it is better to think of 135 as 1 hundred, 3 tens, and 5 ones or as 13 tens and 5 ones. If the children have difficulty, write

$$\begin{array}{r} 100 + 30 + 5 \\ \underline{-\quad (40 + 2)} \end{array} \qquad \begin{array}{r} 13 \text{ tens } 5 \text{ ones} \\ -\ \underline{(4 \text{ tens } 2 \text{ ones})} \end{array}$$

Place examples like the following on the chalkboard:

$$\begin{array}{r} 298 \\ \underline{-\ \ 82} \end{array} \qquad \begin{array}{r} 456 \\ \underline{-\ 19} \end{array}$$

Ask if renaming is necessary. Discuss with children the possibility of subtracting the hundreds first rather than the ones.

Pupil's book, pages 246-247:

These pages may be used for independent work.

Addition: Renaming Ten Ones as One Ten

Use the form which is best for you.

39 + 27 = 66

45 + 28 = 73

56 + 38 = 94

66 + 29 = 95

42 + 49 = 91

38 + 27 = 65

37 + 26 = 63

35 + 47 = 82

19 + 21 = 40

Subtraction: Renaming One Ten as Ten Ones

Use the form that is best for you.

80 - 47 = 33	65 - 28 = 37	52 - 35 = 17
73 - 26 = 47	44 - 17 = 27	86 - 79 = 7
61 - 34 = 27	87 - 29 = 58	38 - 19 = 19

- Continue discussion

To better understand the relationship between addition and subtraction, the children have been helped to see that:

(1) If one number is added to another (doing), then

(2) undoing would be subtracting the same number that was added.

Example: (1) 4 + 2 = 6 (2) 6 - 2 = 4

or

(1) If one number is subtracted from another (doing), then

(2) undoing would be adding the same number that was subtracted.

Example: (1) 6 - 1 = 5 (2) 5 + 1 = 6

This lesson reviews these ideas and then presents "undoing" as a check for addition and subtraction examples.

This review might be motivated by using a problem situation.

Charles and Pat were playing with their toy cars. They wanted to make a garage. Charles had a set of 75 red blocks and Pat had a set of 86 green blocks. They decided to put their blocks together and make a good-sized garage. Charles picked up his box of blocks and dumped them with the pile of blocks Pat had on the floor.

If we wanted to make a record of what was done, who will tell me what to write on the chalkboard?

(1)

$$\begin{array}{r} 86 \\ +\ 75 \\ \hline 161 \end{array}$$

After the boys finished playing, what would Charles have to do? (Separate the blocks so he could take his blocks home.)

What would we write?

(2)

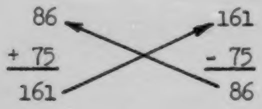
```
  161
 - 75
   86
```

What did we do in our first example?

(Added 75 to 86.)

In the second example, we had to undo what had been done. How did we do this? (Subtracted 75 from 161.)

We can use the idea of "undoing" to check our work.

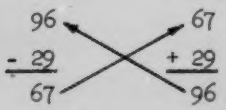


Ask the children to explain what was done. Be sure they understand that if 75 is added to 86, then to undo what has been done, 75 must be subtracted from 161.

Could we use this idea of "undoing" to check subtraction examples? (Yes.)

Would we use subtraction as the check? (No, addition.)

- Write on the chalkboard



Ask the children to explain what was done. Be sure they understand that if 29 is subtracted from 96, then to undo what has been done, 29 must be added to 67.

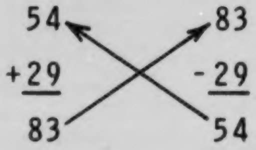
Give several other such examples in addition and subtraction and ask various children to write the check on the chalkboard.

Pupil's book, pages 248 and 249

These pages may be used for independent study. Pupil's book pages 250-253: These pages are for practice. All computations should be checked.

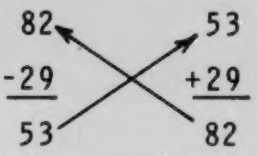
Using Doing and Undoing

Check these examples.

 $\begin{array}{r}54\\+29\\\hline 83\end{array}$ $\begin{array}{r}83\\-29\\\hline 54\end{array}$	$\begin{array}{r}46\\+27\\\hline 73\end{array}$ $\begin{array}{r}73\\-27\\\hline 46\end{array}$
$\begin{array}{r}38\\+59\\\hline 97\end{array}$ $\begin{array}{r}97\\-59\\\hline 38\end{array}$	$\begin{array}{r}27\\+54\\\hline 81\end{array}$ $\begin{array}{r}81\\-54\\\hline 27\end{array}$
$\begin{array}{r}78\\+16\\\hline 94\end{array}$ $\begin{array}{r}94\\-16\\\hline 78\end{array}$	$\begin{array}{r}66\\+29\\\hline 95\end{array}$ $\begin{array}{r}95\\-29\\\hline 66\end{array}$
$\begin{array}{r}54\\+38\\\hline 92\end{array}$ $\begin{array}{r}92\\-38\\\hline 54\end{array}$	$\begin{array}{r}46\\+24\\\hline 80\end{array}$ $\begin{array}{r}80\\-24\\\hline 56\end{array}$ ✓
$\begin{array}{r}43\\+47\\\hline 90\end{array}$ $\begin{array}{r}90\\-47\\\hline 43\end{array}$	$\begin{array}{r}28\\+43\\\hline 71\end{array}$ $\begin{array}{r}71\\-43\\\hline 28\end{array}$

Using Doing and Undoing

Check these examples.

82 -29 53	53 +29 82	42 -17 25	25 +17 42
74 -38 36	36 +38 74	57 -29 28	28 +29 57
96 -59 37	37 +59 96	85 -26 69	69 +26 95 ✓
93 -28 65	65 +28 93	65 -28 37	37 +28 65
41 -27 14	14 +27 41	77 -28 49	49 +28 77

Using Doing and Undoing

Use the form which is best for you. Check your answers.

79 + 86 = 165	84 + 57 = 141	61 + 89 = 150
95 + 37 = 132	86 + 87 = 173	54 + 65 = 119
46 + 95 = 141	59 + 75 = 134	32 + 79 = 111

Using Doing and Undoing

Use the form which is best for you. Check your answers.

284 + 395 = 679	364 + 275 = 639	582 + 377 = 959
473 + 435 = 908	650 + 250 = 900	625 + 284 = 909
360 + 279 = 639	234 + 592 = 826	767 + 142 = 909

Using Doing and Undoing

Compute. Check your answers.

268 - 194 = 74	309 - 135 = 174	537 - 283 = 254
739 - 356 = 383	826 - 472 = 354	905 - 653 = 252
849 - 295 = 554	737 - 584 = 153	606 - 274 = 332

Using Doing and Undoing

Compute. Check your answers.

482 -25 457	158 -93 65	407 + 517 924
461 +38 499	796 - 238 558	561 + 328 889
237 -162 75	709 + 226 935	492 - 233 259

Using Doing and Undoing

Compute. Check your answers.		
46 +99 145	98 -39 59	178 - 83 95
561 -233 328	87 + 38 125	573 +428 1001
725 -392 333	81 -27 54	635 +219 854

V-3. Finding the sum of three numbers

Objective: To review and extend the use of the associative property of addition for whole numbers.

Vocabulary: (No new words.)

Materials: 19 books arranged in stacks of 7, 3, and 9 respectively.

Suggested Procedure:

Begin with a problem such as: On Monday, a teacher brought 7 books to school for the library. On Tuesday she brought 3 books. The following day she brought 9 more. How many books did she bring for the library?

Have children bring the books, a stack at a time, first the 7, then the 3, from the back of the room. Join the second set to the first set. Ask the children how many members are in the union. Then join the third set to the union of the first two sets and identify the number of members in the union. Record on the chalkboard:

$$(7 + 3) + 9 = 19$$

Replace the books in stacks and have the stacks brought again. This time join the set of 9 books to the set of 3 books. Identify the number of members in the union of the two sets. Join the members in the union to the set of 7 books. Record on the chalkboard:

$$7 + (3 + 9) = 19.$$

Ask the children which form they prefer.

$$(7 + 3) + 9 = 19 \quad \text{or} \quad 7 + (3 + 9) = 19$$

Perhaps the first equation will be chosen but in any case call attention to the fact that the sums are the same.

- Recall the commutative property of addition and illustrate through set manipulation that its use leads to other groupings such as:

$(3 + 7) + 9 = 19$ $\qquad$ $7 + (9 + 3) = 19$

$9 + (7 + 3) = 19$ $\qquad$ $(3 + 9) + 7 = 19$

(Note that the equation $(7 + 9) + 3 = 19$ could not be obtained without using both the associative and commutative properties of addition.)

Provide oral practice in adding 3 numbers, each less than 10.

Pupil's book, page 255:

This page provides additional practice in adding three numbers.

Computing the Sum of Three Numbers

3 + 6 + 5 = __14__

2 + 9 + 6 = __17__

5 + 6 + 4 = __15__

8 + 6 + 5 = __19__

5 + 7 + 4 = __16__

3 + 4 + 9 = __16__

7	5	6	9	8	3
2	3	5	4	7	8
8	7	7	5	4	6
17	15	18	18	19	17

1	2	8	6	4	9
7	7	9	3	3	5
9	8	2	5	9	5
17	17	19	14	16	19

4	5	7	9	1	6
9	6	3	8	8	8
4	8	9	2	9	5
17	19	19	19	18	19

3	4	5	7	9	8
8	8	9	5	6	5
5	6	4	7	3	4
16	18	18	19	18	17

● Continue discussion

At this point (after pupils have encountered some of the difficulties inherent in column addition) it seems wise to review what has been commonly known as "higher decade addition"; i.e., the addition of a whole number less than 10 to a number between 10 and 100. Here we are emphasizing those examples in which the sum is not in the same decade as the whole number between 10 and 100.

Write on the chalkboard:

$$\begin{array}{rrrrrrrr} 6 & 16 & 26 & 36 & 46 & 56 & 66 & 76 \\ \underline{+\ 5} & \underline{+\ 5} & \underline{+\ 5} & \underline{+\ 5} & \underline{+\ 5} & \underline{+\ 5} & \underline{+\ 5} & \underline{+\ 5} \\ 11 & & & & & & & \end{array}$$

Ask the children if they know a quick way of finding the sum of 16 and 5, 26 and 5, 36 and 5, etc. In the discussion bring out the fact that if we know $6 + 5 = 11$, then the other sums will have a one in ones place and the number of tens will be one more.

Continue with several other such examples. Try to ascertain if pupils are applying this principle or if they are thinking in terms of renaming ("carrying"). The desired end is instant recognition of the sum.

Pupil's book, pages 256-258:

These pages may be used for practice.

Using Basic Facts

4 + 3 = 7
14 + 3 = 17
24 + 3 = 27
34 + 3 = 37

7 + 8 = 15
17 + 8 = 25
27 + 8 = 35
37 + 8 = 45

4 + 7 = 11
14 + 7 = 21
44 + 7 = 51
34 + 7 = 41

6 + 9 = 15
16 + 9 = 25
56 + 9 = 65
36 + 9 = 45

4 + 8 = 12
14 + 8 = 22
44 + 8 = 52
24 + 8 = 32

5 + 9 = 14
25 + 9 = 34
15 + 9 = 24
35 + 9 = 44

2 + 6 = 8
12 + 6 = 18
52 + 6 = 58
22 + 6 = 28

8 + 8 = 16
48 + 8 = 56
28 + 8 = 36
68 + 8 = 76

3 + 9 = 12
23 + 9 = 32
63 + 9 = 72
13 + 9 = 22

Find the sums

8	5	8	5	9	7
9	9	8	7	7	7
8	7	9	6	9	7
25	21	25	18	25	21

6	9	9	7	5	6
8	5	8	9	7	9
8	7	5	9	6	9
22	21	22	25	18	24

Using Basic Facts

5 + 7 = 12
25 + 7 = 32
15 + 7 = 22

3 + 5 = 8
13 + 5 = 18
33 + 5 = 38

6 + 8 = 14
36 + 8 = 44
16 + 8 = 24

2 + 9 = 11
12 + 9 = 21
32 + 9 = 41

6 + 7 = 13
46 + 7 = 53
16 + 7 = 23

4 + 5 = 9
14 + 5 = 19
24 + 5 = 29

8 + 4 = 12
28 + 4 = 32
18 + 4 = 22

5 + 8 = 13
35 + 8 = 43
15 + 8 = 23

7 + 9 = 16
27 + 9 = 36
17 + 9 = 26

Find the sums.

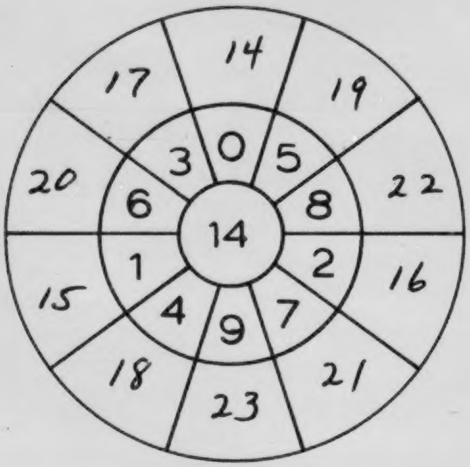
6	7	9	8	9	5
7	8	9	7	7	7
5	+8	4	7	8	9
18	23	22	22	24	21

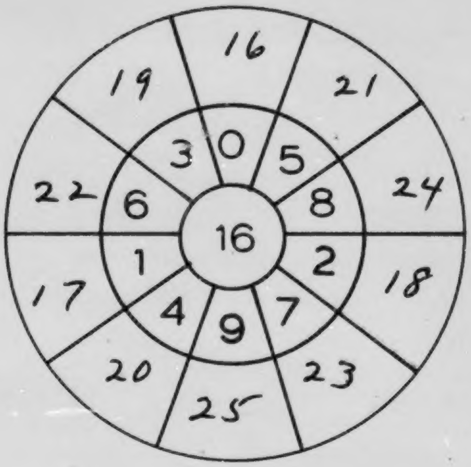
8	5	6	8	7	9
8	8	6	8	7	8
7	5	9	+8	5	9
23	18	21	24	19	26

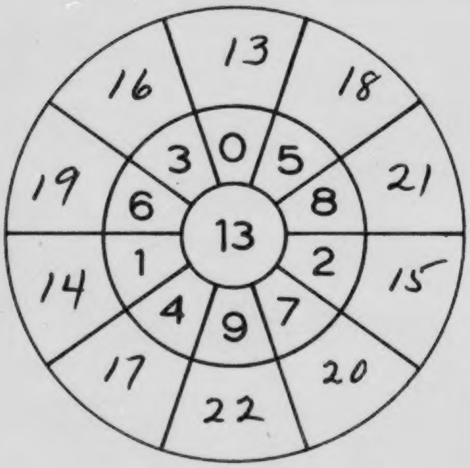
Reviewing Basic Facts

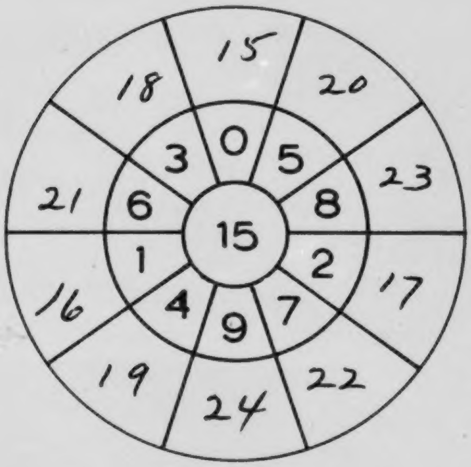
To the number named in the center, add each number named in the second ring.

Write their sum in the outer ring.


14
0
5
8
2
7
9
4
1
6
3
14
19
22
16
21
23
18
15
20
17


16
0
5
8
2
7
9
4
1
6
3
16
21
24
18
23
25
20
17
22
19


13
0
5
8
2
7
9
4
1
6
3
13
18
21
15
20
22
17
14
19
16

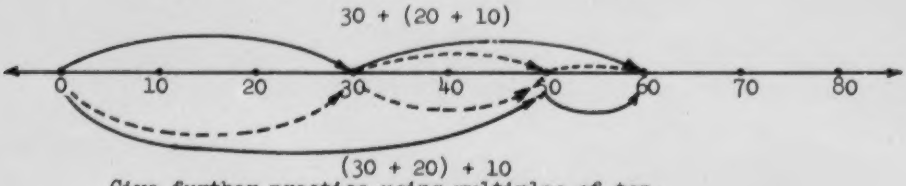

15
0
5
8
2
7
9
4
1
6
3
15
20
23
17
22
24
19
16
21
18

- Continue discussion

Ask what the result would be in an example like:

$$30 + 20 + 10 = \underline{\qquad}$$

Illustrate on the number line that different groupings of 30, 20 and 10 result in the same sum.



Give further practice using multiples of ten.

Now write $32 + 41 + 23 = \underline{\qquad}$ on the chalkboard.

Have children write equations with parentheses to illustrate different ways of grouping these three numbers. Some children may show their work in the following way:

$$(32 + 23) + 41 = 96$$

$$\begin{array}{r} 32 \\ +\ 23 \\ \hline 55 \end{array} \qquad\qquad \begin{array}{r} 55 \\ +\ 41 \\ \hline 96 \end{array}$$

Ask if anyone can suggest a shorter way.

Various forms may be suggested by the children, such as:

$$\begin{array}{lllll} 32 & = & 3 \text{ tens} & 2 \text{ ones} & \\ 41 & = & 4 \text{ tens} & 1 \text{ one} & \\ 23 & = & \underline{2 \text{ tens}} & \underline{3 \text{ ones}} & \\ & & 9 \text{ tens} & 6 \text{ ones} & = 96 \end{array} \qquad\qquad \begin{array}{l} 32 = 30 + 2 \\ 41 = 40 + 1 \\ 23 = \underline{20 + 3} \\ \quad\ \ 90 + 6 = 96 \end{array}$$

Shorter forms that may be suggested are:

$$\begin{array}{rcrcr} 32 & \qquad & 32 & \qquad & 32 \\ 41 & & 41 & & 41 \\ \underline{23} & & \underline{23} & & \underline{23} \\ 90 & & 6 & & 96 \\ \underline{6} & & \underline{90} & & \\ 96 & & 96 & & \end{array}$$

Help children realize that for any of these forms, the grouping of the numbers will not affect the sum.

Pupil's book, page 259:

This page provides practice in finding the sum of three numbers without renaming ones.

● If the class seems to have little difficulty in this sort of addition, show an example like:

$$\begin{array}{r} 24 \\ 35 \\ \underline{15} \end{array}$$

(This extends the idea of finding the sum of three numbers where renaming the ones is necessary.)

You may wish to use this form first:

$$\begin{array}{rcl} 24 & = & 20 + 4 \\ 35 & = & 30 + 5 \\ \underline{15} & = & \underline{10 + 5} \\ & & 60 + 14 = 70 + 4 = 74 \end{array}$$

This algorithm may be helpful to some of the class:

$$\begin{array}{rcr} 24 & & 24 \\ 35 & & 35 \\ \underline{15} & \text{or} & \underline{15} \\ 60 & & 14 \\ \underline{14} & & \underline{60} \\ 74 & & 74 \end{array}$$

Computing the Sum of Three Numbers

If you have to use a longer form, write these examples on another paper.

36	45	16	45
21	24	72	12
$\underline{32}$	$\underline{30}$	$\underline{11}$	$\underline{32}$
89	99	99	89
84	26	43	73
10	43	12	5
$\underline{5}$	$\underline{30}$	$\underline{44}$	$\underline{21}$
99	99	99	99
4	51	66	34
63	26	10	21
$\underline{22}$	$\underline{12}$	$\underline{23}$	$\underline{44}$
89	89	99	99
25	13	67	35
42	64	12	13
$\underline{32}$	$\underline{22}$	$\underline{20}$	$\underline{41}$
99	99	99	89
73	12	24	61
15	54	22	23
$\underline{11}$	$\underline{32}$	$\underline{33}$	$\underline{12}$
99	98	79	96

The children who are able should, of course, be encouraged to use a short form and accomplish the renaming either with or without the help of the small numeral 1 above the tens column.

$$\begin{array}{r} \overset{1}{2}4 \\ 35 \\ \underline{15} \\ 74 \end{array}$$

Pupil's book, pages 260 - 261:

These pages provide practice in finding the sum of three numbers where renaming ones is required.

Computing the Sum of Three Numbers

86 75 24 185	45 27 96 168	79 15 65 159	57 79 22 158
27 22 46 95	64 75 27 166	18 46 23 87	25 49 17 91
48 67 24 139	59 74 56 189	27 97 45 169	63 89 44 196
41 79 98 218	58 99 78 235	78 58 88 224	86 56 66 208

Computing the Sum of Three Numbers

68 18 59 145	46 18 69 133	37 16 99 152	25 88 68 181
17 48 96 161	25 59 89 173	59 98 87 244	14 99 40 153
67 47 68 182	56 29 89 174	45 98 79 222	32 99 88 219
13 99 75 187	84 97 96 277	39 75 79 193	58 96 77 231

V-4. Adding, with renaming more than once

Objective: To extend the use of a short form to situations that require more than one renaming.

Vocabulary: (No new words.)

Suggested Procedure:

On the chalkboard write $498 + 453 = \_\_\_\_$. Work the example in the form below and ask children if they will need to rename the ones? (Yes.) The tens? (Yes.)

$$\begin{array}{l} 400 + 90 + 8 \\ \underline{400 + 50 + 3} \\ 800 + 140 + 11 = (800 + 100) + (40 + 10) + 1 \\ \qquad\qquad\qquad\;\; = 951 \end{array}$$

Beside this computation, show the same example in this form:

$$\begin{array}{rl} 498 & \\ \underline{+\ 453} & \\ 11 & \text{(sum of 8 ones and 3 ones)} \\ 140 & \text{(sum of 9 tens and 5 tens)} \\ \underline{800} & \text{(sum of 4 hundreds and 4 hundreds)} \\ 951 & \end{array}$$

Show that with these forms, it does not matter whether the adding of the ones is done first or last. Next, write the example again.

$$\begin{array}{r} 498 \\ \underline{+\ 453} \end{array}$$

Ask a child to explain how the computation is done using a shorter form, such as

$$\begin{array}{r} {\scriptstyle 11} \\ 498 \\ \underline{453} \\ 951 \end{array}$$

Ask him why it is helpful to begin by adding the ones.

- Present the example 236 + 669 = ____ and write the form used for computing. Ask the children if the ones will have to be renamed. (Yes.) Point out that it does not appear that the tens will have to be renamed since 3 tens + 6 tens = 9 tens. Use the expanded form first,

$$\begin{array}{r} 200 + 30 + 6 \\ \underline{600 + 60 + 9} \\ 800 + 90 + 15 \end{array}$$

Ask how children would add 90 and 15. Suggest that this is 90 + (10 + 5) or (90 + 10) + 5 or 100 + 5. So, 800 + 100 + 5 = 905. Was it necessary to rename tens as one hundred? (Yes.)

Rewrite the example in this way:

$$\begin{array}{r} 236 \\ \underline{+\ 669} \\ 15 \\ 90 \\ \underline{800} \\ 905 \end{array}$$

Then ask if someone can show a shorter form, such as

$$\begin{array}{r} {\scriptstyle 11} \\ 236 \\ \underline{+\ 669} \\ 905 \end{array}$$

Have children work the following examples at the chalkboard, using whichever method they prefer:

357 + 546 = ________
729 + 171 = ________
415 + 287 = ________
698 + 223 = ________
562 + 389 = ________

This work may be extended for more able pupils to include the addition of numbers greater than 1000.

Pupil's book, pages 262-269

These pages provide opportunity for individual practice.

Pupil's book, pages 270-271

These pages may be used by children who will profit from such practice.

The Cross-Number Puzzle, *Pupil's book, pages 272-273* may be done by all children.

Computing Sums

Find the sum of 497 and 353. Use the form which is best for you.

$$\begin{array}{r} 400 + 90 + 7 \\ \underline{300 + 50 + 3} \\ 700 + 140 + 10 = 850 \end{array} \qquad \begin{array}{r} 497 \\ \underline{+353} \\ 10 \\ 140 \\ \underline{700} \\ 850 \end{array} \qquad \begin{array}{r} 497 \\ \underline{353} \\ 850 \end{array}$$

587 + 267 = 854

$$\begin{array}{r} 587 \\ \underline{267} \end{array}$$

338 + 379 = 717

$$\begin{array}{r} 338 \\ \underline{379} \end{array}$$

468 + 85 = 553

$$\begin{array}{r} 468 \\ \underline{85} \end{array}$$

287 + 486 = 773

$$\begin{array}{r} 287 \\ \underline{486} \end{array}$$

653 + 298 = 951

$$\begin{array}{r} 653 \\ \underline{298} \end{array}$$

447 + 379 = 826

$$\begin{array}{r} 447 \\ \underline{379} \end{array}$$

526 + 298 = 824

$$\begin{array}{r} 526 \\ \underline{298} \end{array}$$

437 + 78 = 515

$$\begin{array}{r} 437 \\ \underline{78} \end{array}$$

383 + 598 = 981

$$\begin{array}{r} 383 \\ \underline{598} \end{array}$$

Computing Sums

964 + 89 = 1053 964 89	178 + 785 = 963 178 785
457 + 388 = 845 457 388	169 + 765 = 934 169 765
654 + 297 = 951 654 297	586 + 378 = 964 586 378
264 + 579 = 843 264 579	736 + 197 = 933 736 197

Computing Sums

264 + 579 = 843

736 + 197 = 933

654 + 297 = 951

586 + 278 = 864

964 + 89 = 1053

178 + 785 = 963

457 + 388 = 845

169 + 765 = 934

Computing Sums

343 +379 722	154 + 767 921
316 + 594 910	899 + 21 920
245 +487 732	487 +397 884
568 +258 826	654 +279 933

Computing Sums

148 +594 742	727 +688 1415
546 +356 902	225 +599 824
676 +279 955	388 +167 555
368 +599 967	479 +187 666

Computing Sums

346 +475 821	347 +453 800
349 +499 848	561 +299 860
492 +369 861	389 +148 537
344 +297 641	476 +224 700

Computing Sums

(1) 638 + 393 = 1031

(2) 859 + 584 = 1443

(3) 747 + 679 = 1426

(4) 635 + 779 = 1414

(5) 235 + 898 = 1133

(6) 999 + 341 = 1340

(7) 766 + 398 = 1164

(8) 726 + 775 = 1501

(9) 984 + 16 = 1000

(10) 585 + 656 = 1241

(11) 737 + 287 = 1024

(12) 539 + 898 = 1437

(13) 632 + 989 = 1621

(14) 726 + 787 = 1513

(15) 315 + 697 = 1012

(16) 834 + 476 = 1310

(17) 347 + 653 = 1000

(18) 234 + 277 = 511

(19) 564 + 236 = 800

(20) 298 + 345 = 643

(21) 325 + 297 = 622

(22) 248 + 398 = 646

(23) 576 + 297 = 873

(24) 469 + 331 = 800

(25) 345 + 287 = 632

(26) 573 + 198 = 771

(27) 455 + 555 = 1010

(28) 423 + 298 = 721

(29) 827 + 173 = 1000

(30) 769 + 199 = 968

Order Relations

Use the symbol < (less than) or > (greater than).

$35 + 63 \boxed{<} 100$

$485 + 314 \boxed{<} 800$

$500 + 600 \boxed{>} 1000$

$86 + 97 \boxed{<} 200$

$54 + 55 \boxed{>} 100$

$300 + 263 \boxed{>} 500$

$499 + 99 \boxed{<} 600$

$54 + 26 \boxed{>} 70$

$437 + 463 \boxed{<} 1000$

$160 + 395 \boxed{>} 500$

$200 + 341 \boxed{>} 500$

$555 + 461 \boxed{>} 1000$

$37 + 67 \boxed{>} 100$

$297 + 312 \boxed{>} 600$

$144 + 50 \boxed{<} 200$

Computing Sums

6483 +1279 7762	3688 +2285 5973	4479 +5396 9875
3327 +6599 9926	7248 +2476 9724	1532 + 6289 7821
2448 + 469 2917	3446 + 6377 9823	3584 + 4278 7862

Computing Sums

278 434 276 988	385 287 153 825	446 337 144 927	639 128 193 960
166 399 415 980	372 584 21 977	600 37 288 925	392 303 205 900
343 167 472 982	666 118 75 859	747 89 155 991	259 496 225 980
456 125 397 978	247 198 457 902	483 257 169 909	578 65 265 908

A Cross-Number Puzzle

^{1}3	^{2}8	^{3}5	▨	▨	^{4}5	^{5}3	^{6}9
^{7}4	1	▨	^{8}2	^{9}6	▨	^{10}6	9
0	▨	^{11}4	6	1	^{12}7	▨	9
▨	^{13}2	1	▨	▨	^{14}2	^{15}7	▨
▨	^{16}3	0	▨	▨	^{17}8	2	▨
^{18}1	▨	^{19}3	^{20}5	^{21}6	9	▨	^{22}2
^{23}5	2	▨	^{24}1	2	▨	^{25}1	7

Across

1. Three hundred eighty-five
4. Another way to write 500 + 30 + 9
7. 1 more than 3 tens and 10 ones.
8. 8 + 8 + 8 + 2
10. (60 + 3) + 3 + 3
11. 46 hundreds and 17 ones
13. 31 - 10
14. 9 + 9 + 9
16. 630 - 600
17. _____ is 12 more than 70.
19. 3000 + 500 + 60 + 9
23. 539 = _____ tens + 19 ones
24. Months in a year
25. 26 - 9

Down

1. 34 tens
2. _____ < 82
5. 556 - 520
6. Largest number less than 1000.

Down

8. 2 tens and 6 ones
9. 33 + 28
11. Four thousand one hundred three
12. 4289, 5289, 6289, ________
13. 1 ten and 13 ones
15. 720 equals how many tens?
18. 3 fives
20. 70 - 19
21. 5 tens and 12 ones
22. 7, 12, 17, 22, ______

V-5. Subtracting with renaming more than once

Objective: To extend use of a short form to subtraction situations requiring more than one renaming.

Vocabulary: (No new words.)

Suggested Procedure:

Write on the chalkboard 963 - 478 = ____, and below write the example in a form for computing. Show the use of the expanded form when only one renaming is accomplished at a time:

$$\begin{array}{r} 900 + 60 + 3 \\ \underline{-(400 + 70 + 8)} \end{array} \qquad \begin{array}{r} 900 + 50 + 13 \\ \underline{-(400 + 70 + 8)} \end{array} \qquad \begin{array}{r} 800 + 150 + 13 \\ \underline{-(400 + 70 + 8)} \\ 400 + 80 + 5 = 485 \end{array}$$

Ask whether it might not have been possible to accomplish the renaming in fewer steps.

Now suggest that 963 be thought of as 9 hundreds, 6 tens and 3 ones, and write:

$$\begin{array}{r} 963 \\ \underline{-\ 478} \end{array} \qquad \begin{array}{rlrlrl} & 9 & \text{hundreds} & 6 & \text{tens} & 3 \ \text{ones} \\ - & \underline{4} & \underline{\text{hundreds}} & \underline{7} & \underline{\text{tens}} & \underline{8 \ \text{ones}} \end{array}$$

Show that the renaming can be written the following way, saying,

> We will have to rename 1 ten as 10 ones. Let's do that and write:
>
> $$\begin{array}{rlrlrl} & 9 & \text{hundreds} & \overset{5}{\cancel{6}} & \text{tens} & \overset{13}{\cancel{3}} \ \text{ones} \\ - & \underline{4} & \underline{\text{hundreds}} & \underline{7} & \underline{\text{tens}} & \underline{8 \ \text{ones}} \end{array}$$
>
> We will have to rename one of the hundreds as 10 tens. We write:
>
> $$\begin{array}{rlrlrl} & \overset{8}{\cancel{9}} & \text{hundreds} & \overset{15}{\cancel{6}} & \text{tens} & \overset{13}{\cancel{3}} \ \text{ones} \\ - & \underline{4} & \underline{\text{hundreds}} & \underline{7} & \underline{\text{tens}} & \underline{8 \ \text{ones}} \end{array}$$

and subtract.

Begin the subtraction at the ones place, and write the answer as 4 hundreds, 8 tens, 5 ones below the line. Ask a child what the decimal numeral is, and write 485. Ask whether it was necessary to have the words "hundreds, tens, ones" written down in order to remember what we were doing.

Show the use of the short form, and point out the fact that it is not necessary to do all the renamings before starting to compute. Rather, renaming may be done one column at a time as needed.

$$\begin{array}{r} 963 \\ \underline{-\ 478} \end{array} \qquad \begin{array}{r} 9\overset{5}{\cancel{6}}\overset{13}{\cancel{3}} \\ \underline{-\ 478} \end{array} \qquad \begin{array}{r} \overset{8}{\cancel{9}}\overset{15}{\cancel{6}}\overset{13}{\cancel{3}} \\ \underline{-\ 478} \end{array}$$

If children are able to rename mentally, without "reminder" numerals, they should be encouraged to do so; on the other hand, the "helps" should not be actively discouraged and children who still need to use the expanded form should feel free to continue to do so.

Ask children to look at the example:

$$\begin{array}{r} 786 \\ \underline{-\ 389} \end{array}$$

It is easy to see that renaming one ten as 10 ones will be necessary, but it takes more foresight than is to be expected of many children to anticipate the need for renaming one hundred as 10 tens. Therefore, ask that the expanded form be used.

Show that renaming a hundred as 10 tens is necessary, where it did not appear to be at the beginning.

$$\begin{array}{r} 600 + 170 + 16 \\ \underline{-\ (300 + \ \ 80 + \ \ 9)} \\ 300 + \ \ 90 + \ \ 7 \end{array}$$

Use the short form, and show that again, beginning at the right, it is not necessary to do all the renaming at once.

$$\begin{array}{rccc} & 6 & 17 & 16 \\ & \cancel{7} & \cancel{8} & \cancel{6} \\ - & 3 & 8 & 9 \\ \hline \end{array}$$

Use other examples. Have a child rewrite the example in vertical form and show how to use the expanded form to obtain the answer. Have another child use the short form.

Pupil's book, pages 274-276:

These pages may be used for independent work.

Computing Differences

532 -275 257	983 -271 712
816 -387 429	614 -389 225
721 -289 432	435 -263 172
945 -467 478	736 -449 287

Computing Differences

825 - 376 449	254 - 189 65
334 - 167 167	513 - 286 227
622 - 235 387	324 - 58 266
461 - 378 83	671 - 298 373

Computing Differences

651 - 287 = 364

427 - 269 = 158

821 - 367 = 454

745 - 479 = 266

653 - 296 = 357

514 - 395 = 119

336 - 278 = 58

534 - 298 = 236

● Continue discussion

Since subtracting from numbers such as 400, 4000, 302, etc. may involve a different type of thinking for efficient renaming, these kinds of examples are given special attention.

While it is true that 400 - 268 = ____ can be computed using this short form,

$$\begin{array}{r} \overset{39}{\cancel{40}}{}^{1}0 \\ -\ 268 \\ \hline 132 \end{array}$$

only one renaming is necessary if 400 is thought of as 40 tens.

$$\begin{array}{r} \overset{39}{\cancel{40}}{}^{1}0 \\ -\ 268 \\ \hline 132 \end{array}$$

In the example above, one of the 40 tens has been renamed 10 ones. The expanded form would be:

$$\begin{array}{r} 390 + 10 \\ -\ (260 + \ 8) \\ \hline 130 + \ 2 \ = \ 132 \end{array}$$

Necessarily, then, the lesson should begin with a review of thinking about numbers as tens and ones.

302 = 30 tens 2 ones
400 = 40 tens 0 ones
208 = 20 tens 8 ones

Follow these types of renamings with:

500 = 49 tens 10 ones
600 = 59 tens 10 ones
700 = 69 tens 10 ones
408 = 39 tens 18 ones
402 = 39 tens 12 ones
705 = 69 tens 15 ones

Then write: 300 - 152 on the chalkboard. Ask the children to compute the difference. A possible form is:

$$\begin{array}{rl} 300 = & 200 + 90 + 10 \\ 152 = & \underline{-(100 + 50 + \ \ 2)} \end{array}$$

Suggest that both renamings could be accomplished by thinking of 300 as 30 tens. See if the children can show the renaming of 300 as 290 + 10. If no one can do this, rewrite the example.

$$\begin{array}{r} 300 \\ \underline{-\ 152} \end{array}$$

> If we think of 300 as 30 tens, then we can rename 300 as 29 tens 10 ones. What is the numeral for 29 tens? (290.) Then couldn't we rename 300 as 290 + 10? (Yes.)
>
> In expanded form the example would look like this:
>
> $$\begin{array}{r} 290 + 10 \\ \underline{-(150 + \ \ 2)} \\ 140 + \ \ 8 = 148 \end{array}$$
>
> Can anyone show us a shorter form?
>
> $$\begin{array}{r} \overset{29}{\cancel{30}}\overset{1}{0} \\ \underline{-\ 152} \\ 148 \end{array}$$

Several other examples of the same type should be computed using the new forms.

Pupil's book, pages 277-280:

These pages may be used for practice.

Using Renaming with Subtraction

1) 400 = 40 tens
400 = 39 tens 10 ones
400 = 390 + 10

$$\begin{array}{r}400\\-187\\\hline 213\end{array}$$

2) 500 = 50 tens
500 = 49 tens 10 ones
500 = 490 + 10

$$\begin{array}{r}500\\-132\\\hline 368\end{array}$$

3) 600 = 60 tens
600 = 59 tens 10 ones
600 = 590 + 10

$$\begin{array}{r}600\\-249\\\hline 351\end{array}$$

4) 700 = 70 tens
700 = 69 tens 10 ones
700 = 690 + 10

$$\begin{array}{r}700\\-297\\\hline 403\end{array}$$

5) 800 = 80 tens
800 = 79 tens 10 ones
800 = 790 + 10

$$\begin{array}{r}800\\-256\\\hline 544\end{array}$$

6) 900 = 90 tens
900 = 89 tens 10 ones
900 = 890 + 10

$$\begin{array}{r}900\\-776\\\hline 124\end{array}$$

Computing Differences

600 -351 249	904 -498 406
305 - 68 237	703 -265 438
900 -321 579	804 -475 329
702 -376 326	603 -247 356

Computing Differences

784 -292 492	386 - 99 287
304 -291 13	801 -794 7
764 -198 566	368 -129 239
430 -109 321	800 -298 502

Order Relations

Write > or < in the box.

400 - 299 $\boxed{>}$ 100

700 - 304 $\boxed{<}$ 400

500 - 201 $\boxed{<}$ 300

200 - 98 $\boxed{>}$ 100

3286 - 200 $\boxed{>}$ 3000

6457 - 387 $\boxed{>}$ 6000

684 - 80 $\boxed{>}$ 600

500 - 267 $\boxed{<}$ 300

700 - 302 $\boxed{<}$ 400

1000 - 506 $\boxed{<}$ 500

- Continue discussion (optional)

A similar procedure may be used with examples such as: 8000 - 2765 = ______. For efficient renaming, think of 8 thousand as 800 tens. Then rename 800 tens as 799 tens 10 ones. Both the expanded and short forms are given below.

$$\begin{array}{rl} 8000 = & 7990 + 10 \\ 2765 = & \underline{-(2760 + \ 5)} \\ & 5230 + \ 5 = 5235 \end{array} \qquad \begin{array}{r} \overset{799}{\cancel{800}}0 \\ \underline{-\ 2765} \\ 5235 \end{array}$$

Begin the discussion with the children by reviewing these kinds of renamings:

6000 = ______ tens
5000 = ______ tens
2000 = ______ tens
2000 = ______ tens 10 ones
7000 = ______ tens 10 ones
9008 = ______ tens 18 ones

etc.

Follow the same type of procedure as was suggested on pages 472-473 making the necessary adaptations for dealing with thousands rather than hundreds.

Pupil's book, pages 281-285:
These pages provide review of this section.

* Pupil's book, pages 286-314 (Optional) Pages 286-289 afford interesting practice in computing sums. Pages 290-296 lead more able pupils to experiment with subtraction by complementation.

Pupil's book, pages 297-314. These sets of problems may be interspersed with other lessons, to provide practice in writing equations and solving problems.

Using Renaming with Subtraction

1) 7000 = 700 tens
7000 = 699 tens 10 ones
7000 = 6990 + 10

7000
- 2689
4311

2) 5000 = 500 tens
5000 = 499 tens 10 ones
5000 = 4990 + 10

5000
- 1234
3766

3) 2002 = 200 tens 2 ones
2002 = 199 tens 12 ones
2002 = 1990 + 12

2002
- 1679
323

4) 6008 = 600 tens 8 ones
6008 = 599 tens 18 ones
6008 = 5990 + 18

6008
- 2659
3349

5) 4010 = 401 tens 0 ones
4010 = 400 tens 10 ones
4010 = 4000 + 10

4010
- 2605
1405

6) 9075 = 907 tens 5 ones
9075 = 906 tens 15 ones
9075 = 9060 + 15

9075
- 2066
7009

Computing Differences

6400 -2209 4191	8008 -2439 5569
7034 -1027 6007	7041 -2039 5002
6481 -2276 4205	6005 -1896 4109
9800 -2709 7091	3980 -1777 2203

Computing Differences

5721 -2317 3404	8105 -1099 7006
8407 -2308 6099	1827 - 318 1509
5776 -2149 3627	3987 -1259 2728
3605 -2176 1429	9002 -2345 6657

Computing Differences

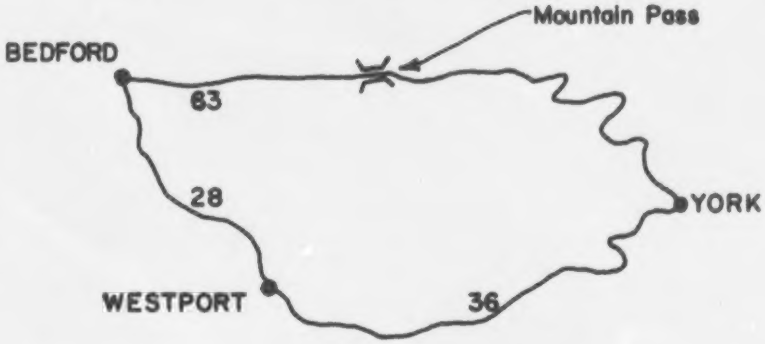
5721 -2598 3123	7814 -3675 4139
9445 -6258 3187	8652 -5387 3265
3268 -1752 1506	7624 -3378 4246
5816 -1749 4067	3730 -2395 1335

Computing Differences

6323 -1278 5045	6401 -2389 4012
4727 -3193 1534	8254 -2128 6126
6845 -2376 4469	7641 -2238 5403
4302 -2164 2138	5728 -1219 4509

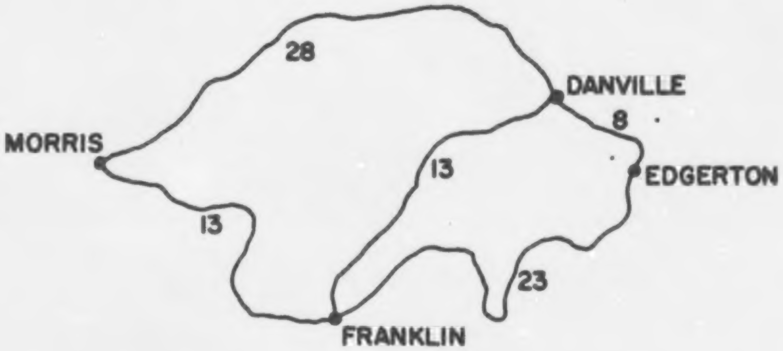
☆ Road maps

1. Here is a road map. The numbers on the roads show distances in miles.


Mountain Pass
BEDFORD
63
28
YORK
WESTPORT
36

Which is the shorter route from Bedford to York? Via Westport or over the pass? Over the pass.

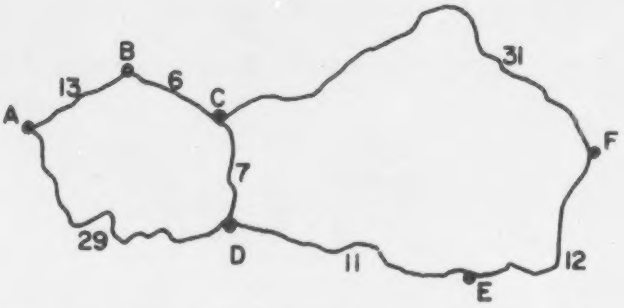
2. Here is another road map.


28
DANVILLE
8
MORRIS
13
EDGERTON
13
23
FRANKLIN

What is the shortest route from Morris to Edgerton?

Via Franklin and Danville

3. What is the shortest route from A to F? ABCDEF


A
B
C
D
E
F
13
6
31
7
29
11
12

How many routes are there from A to F? 4 One of them is A B C F. List all the others.

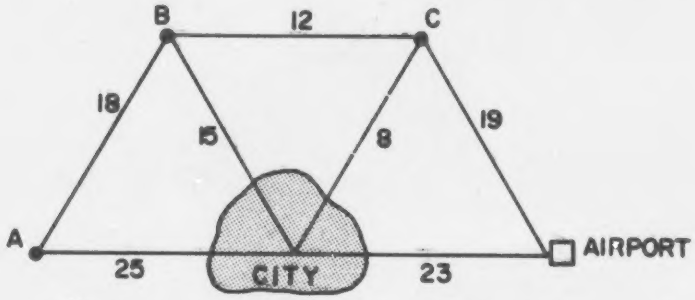
ADEF

ABCDEF

ADCF

Did you check all of these in order to find which one is shortest?

4. Here is a map of a city with some nearby towns and the airport. The numbers show how long it takes in minutes to drive from one place to another:


B
12
C
18
15
8
19
A
25
CITY
23
AIRPORT

What is the quickest way to get from town A to the airport?

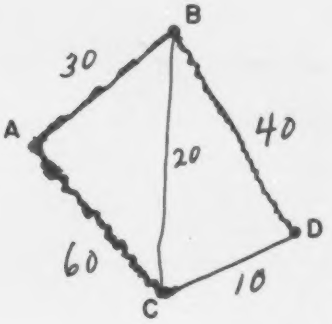
Through the city.

5. Here is a chart showing the distances by direct road between towns A, B, C, D. For example, the distance between A and B is 30 miles. No number was put in the A, D square because there is no direct road from A to D.

	A	B	C	D
A	0	30	60	▨
B	30	0	20	40
C	60	20	0	10
D	▨	40	10	0

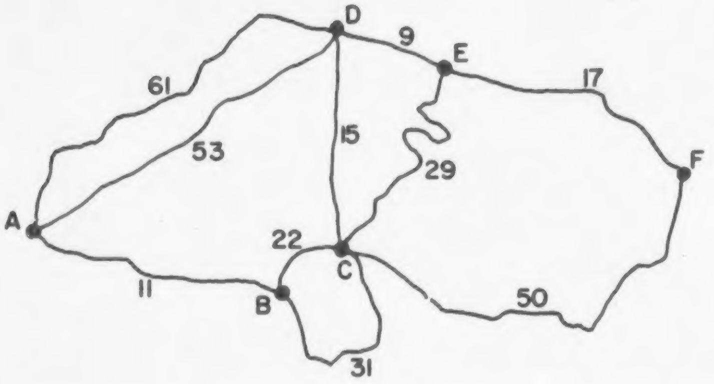
Find the shortest route from A to D. A to B to C to D

Can you draw a map to help you? Here are the towns A, B, C, D:



The road from A to C cannot be straight. Why? Answers will vary.

6. Here is a more complicated map. There are 14 ways to get from A to F. Can you find the shortest way without looking at all 14 ways separately? The questions below will help you.



(1) How far is it from A to C by the shortest route? 33 miles.

(2) How far is it from A to D by the shortest route? (Your answer to question 1 will help you.) 48 miles.

(3) How far is it from A to E by the shortest route? (Your answers to questions 1 and 2 will help you.) 57 miles.

(4) How far is it from A to F by the shortest route? (Your answers to questions 1 and 3 will help you.) 74 miles.

✱ Subtraction by Complementation

In these problems we are going to play with equations a little bit and then, using equations, we will find a new way to do subtraction.

1. Is this equation correct? Yes.

$$7 + 8 = 9 + 6$$

Now we add the same number to both sides:

$$7 + 8 + 4 = 9 + 6 + 4$$

Is the equation still correct? Yes.

2. Is this equation correct ? Yes.

$$12 + 7 = 10 + 9$$

Now we subtract the same number from both sides:

$$12 + 7 - 8 = 10 + 9 - 8$$

Is the equation still correct? Yes.

If you add the same number to both sides of a correct equation, the equation remains correct.

If you subtract the same number from both sides of a correct equation, the equation remains correct.

3. Let's check these rules with some more examples. Do the arithmetic on each side of the equation and put the answer in the blank below.

$$8 + 3 = 15 - 4$$

$$11 = 11$$

Are the two sides the same? Yes. Now add 9 to both sides:

$$8 + 3 + 9 = 15 - 4 + 9$$

$$20 = 20$$

Are the two sides still the same? Yes. N[illegible]e

$$12 + 16 = 13 + 15$$

$$28 = 28$$

Add 7 to both sides:

$$12 + 16 + 7 = 13 + 7 + 15$$

$$35 = 35$$

Notice where we put the 7 on the right side. Does it make any difference where we put it? No.

Now try these:

$$10 + 10 = 13 + 7$$

$$20 = 20$$

Subtract 3 from both sides:

$$10 + 10 - 3 = 13 - 3 + 7$$

$$17 = 17$$

$$45 + 62 = 100 + 7$$

$$107 = 107$$

Subtract 5 from both sides:

$$45 - 5 + 62 = 100 + 7 - 5$$

$$102 = 102$$

$$137 - 18 = 100 + 19$$

$$119 = 119$$

Subtract 100 from both sides:

$$137 - 100 - 18 = 100 - 100 + 19$$

$$19 = 19$$

4. Which of these equation are correct? Use the rules you learned in problems 1 and 2 above.

$9 + 8 = 17$ correct? Yes.

$9 + 8 + 372 = 17 + 372$ correct? Yes.

$14 + 9 + 8 = 17 + 14$ correct? Yes.

$9 + 523 + 8 = 523 + 17$ correct? Yes.

$47 + 18 = 13 + 50$ correct? No.

$47 + 18 - 28 = 13 + 50 - 28$ correct? No.

$55 = 25 + 30$ correct? Yes.

$55 + 100 - 100 = 25 + 30$ correct? Yes.

5. Fill in the blanks:

0 + 9 = 9
1 + 8 = 9
2 + 7 = 9
3 + 6 = 9
4 + 5 = 9

5 + 4 = 9
6 + 3 = 9
7 + 2 = 9
8 + 1 = 9
9 + 0 = 9

The numbers you have just written are called the nine's complements of 1, 2, 3, 4, 5, 6, 7, 8, 9. "Complement" is a word you will be seeing in mathematics. It means "the piece left over". We call 2 the nine's complement of 7 because 2 is the "piece" left over after you subtract 7 from 9. The word "complement" is related to the word "complete." It has nothing to do with "compliment". What does compliment mean?

6. Do these subtractions:

$$\begin{array}{r}9\\-7\\\hline 2\end{array}\qquad\begin{array}{r}99\\-38\\\hline 61\end{array}\qquad\begin{array}{r}999\\-506\\\hline 493\end{array}\qquad\begin{array}{r}9999\\-2914\\\hline 7085\end{array}$$

If you know the nine's complements, subtracting numbers from 9 or 99 of 999, etc., is easy. Tell here how to do it:

Answers will vary.

7. 9 + 1 = 10　　999 + 1 = 1000

99 + 1 = 100　　9999 + 1 = 10,000

8. Are these equations correct? Yes.

1000 - 478 = 999 + 1 - 478

1000 - 478 = 999 - 478 + 1

9. Do these:

999 - 328 = 671

1 + 999 - 328 = 672

1000 - 328 = 672

1000 - 479 = 521

1000 - 165 = 835

10000000 - 4671023 = 5328977

If you know the nine's complements, subtracting numbers from 10 or 100 or 1000, etc., is easy. Tell here how to do it.

Answers will vary.

10. Subtracting 10, 100, 1000, etc., from other numbers is easy, too. Do these:

184 - 100 = 84

1436 - 1000 = 436

2048 - 1000 = 1048

11. Is this equation correct? Yes.

$$723 - 489 = 723 - 489 + 1000 - 1000$$

Let's change the order of the terms on the right.

$$723 - 489 = 1000 - 489 + 723 - 1000$$

Is the equation still correct? Yes.

Now we will use this equation to help us to do the subtraction

$$723 - 489$$

Let's do the right side of the equation:

$1000 - 489 =$ 511

$1000 - 489 + 723 =$ 1234

$1000 - 489 + 723 - 1000 =$ 234

Now do the subtraction the usual way.

$$\begin{array}{r} 723 \\ -\ 489 \\ \hline 234 \end{array}$$

Did you get the same answer? Yes.

12. Here are two ways to do this subtraction:

523 - 297

New way:

1000 - 297 = 703

Add 523 to what you just got. 1226

Subtract 1000 from what you just got. 226

Old way:

523
- 297
226

Did you get the same answer? Yes.

13. Do these subtractions:

New way:		Old way:
1000 - 387 =	613	615
Add 615:	1228	- 387
Subtract 1000:	228	228
1000 - 195 =	805	263
Add 263:	1068	- 195
Subtract 1000:	68	68

Explain why the new way works. Answers will vary.

Set 4

Solving Problems

Write an equation and an answer sentence.

1. During a lesson about Indians, Kevin learned that one chief lived to be 94 years old. He thought that in 85 years he would be that old. What is Kevin's age now?

   Equations may vary.

   Kevin is 9 years old.

2. Julie's family went on a boat trip. They traveled 68 miles to an island and 75 miles back to the harbor. How many miles did they travel during the whole trip?

   They traveled 143 miles.

3. A basketball was on sale for 79¢. Some children wanted to buy it. They had 95¢. How much money would they have left after buying the ball?

   They would have 16¢ left.

4. Mr. Ford has 32 pills in his medicine bottle. If he had 40 pills when he bought the medicine, how many pills has he used so far?

He has used 8 pills.

5. There were 95 boys on the playground. 47 of them were playing on the equipment. How many boys were doing something else?

Forty-eight boys were doing something else.

6. Susan needs 97 cents to buy a doll. She has 36 cents. How much more money does she need?

She needs 61¢ more.

Set 5

Solving Problems

Write an equation and an answer sentence.

1. Jack's small turtle is 26 years old. His large turtle is 65 years old. How much older is the large turtle?

____

The large turtle is 39 years older then the small one.

2. Saturday Mary went to visit her cousin. She went 183 miles on the train. She rode 32 miles from the train station to her cousin's house. How far did Mary ride on her way to her cousin's house?

____

She rode 215 miles.

3. Last year Mary went on a 675-mile trip. How much further did she travel last year than she did on the trip to her cousin's house?

____

She went 460 miles further last year.

4. Mr. Smith had 573 bricks to make a walk. He had 28 left when the walk was finished. How many bricks did he use?

____

He used 545 bricks.

5. Sally counted her steps to school. She went 73 steps to the corner and 28 more to the school yard. How many steps did she take from her house to the school yard?

____

She took 101 steps.

Set 6

Solving Problems

Write an equation and complete the answer sentence for each problem.

1. During the annual Campfire Girls' candy sale, Mary's team sold 232 boxes of mints. Sue's team sold 472 boxes of mints. Find the total number of boxes sold by the teams of the two girls.

   ____

   They sold 704 boxes.

2. The pupils of Oak School collected gifts for other children at Christmas. They collected 133 books and 316 toys. How many gifts were collected?

   ____

   449 gifts were collected.

3. John is 55 inches tall. His father is 74 inches tall. How many inches must John grow to be as tall as his father?

   ____

   John must grow 19 inches.

4. Sue has $25 to buy a bicycle. The bicycle costs $42. How much more money must she save?

   ____

   Sue must save $17.

5. A high school stadium has 700 seats. 462 tickets have been sold for a game. How many tickets are left?

____

238 tickets are left.

6. A bear in a zoo weighs 746 pounds. A seal weighs 572 pounds. How much less does the seal weigh than the bear?

____

The seal weighs 174 pounds less than the bear.

7. Billy has 48 marbles. Tom has 52. How many more marbles does Tom have than Billy?

____

Tom has 4 more marbles than Billy.

8. Write the equation and the answer sentence.

Jimmy had 267 toy cars. If he gave away 82 cars, how many cars would he have then?

____

He would have 185 cars.

Set 7

Solving Problems

Write an equation and the answer sentence.

1. Miss Lane has 200 erasers. She has 34 children in her class. If she gives one eraser to each child in her class, how many erasers does she still have?

   ____

   She still has 166 erasers.

2. The third and fourth grades were going on a field trip. There were 113 children in all. There were 57 children in the fourth grade classes. How many children were in the third grade classes?

   ____

   There were 56 children in third grade classes.

3. Mr. Williamson needs 100 gallons of paint to paint his motel. He has 11 gallons. How many more gallons does he need?

   ____

   He needs 89 gallons more.

4. Michael has 75¢. He wanted to buy a book for 90¢. How much more money does he need?

He needs 15¢ more.

5. During the Wheeler family's first trip they wrote 77 post cards. On their next trip they wrote 39 post cards. How many post cards did they write on both trips?

They wrote 116 postcards in all.

6. There were 23 apples in a basket. Eighteen of the apples were eaten. How many apples were still left in the basket?

5 apples were still in the basket.

Set 8

Write an equation and complete the answer sentence.

1. There are 298 children in school. If 176 of them are girls, how many of them are boys?

   ____

   There are 122 boys.

2. Nan has 13 dolls. Polly has 17 dolls. Beatrice has 9 dolls. How many dolls do the girls have?

   ____

   The girls have 39 dolls.

3. One day there were 98 boats on the river. The next day, after a storm, there were only 37. How many boats were missing on the second day?

   ____

   61 boats were missing.

4. Thirty-one children were invited to Mary's birthday party. Twenty-six came. How many children did not come?

   ____

   5 children did not come.

5. A museum had 372 pictures. Some were stolen. If 297 pictures were left, how many were stolen?

____

75 pictures were stolen.

6. In the library there were 213 books. Some new books were given to the library. Then the library had 300 books. How many books were given to the library?

____

87 books were given to the library.

7. There were 230 animals in the zoo. One hundred seventy-five of them were dangerous. How many of them were not?

____

55 animals were not dangerous.

8. One hundred seventy-two rockets had been sent off. Then 111 more were sent off. How many rockets were fired?

____

283 rockets were fired.

Set 9

Solving Problems

Write an equation and an answer sentence.

1. The corner drug store had 987 ball-point pens in stock. They sold 14 of them. How many do they have now?

   ____

   They have 973 pens now.

2. Mrs. Foster had a lot with 540 peach trees. She bought another lot with 230 more peach trees. How many peach trees does she have now?

   ____

   She has 770 peach trees now.

3. The Lincoln School has a total of 957 children. If 28 of them are in Mrs. Hoff's class, how many are in the rest of the school?

   ____

   929 are in the rest of the school.

4. Claudia is collecting stamps. She has 969 stamps in all. Of these, 702 are not American stamps. How many of them are American stamps?

267 are American stamps.

5. Alfred is reading a book that has 234 pages. He has to read another 41 pages before he finishes the book. How many pages has he read?

He has read 193 pages.

6. Girl Scout Troop 66 sold 596 boxes of cookies. Troop 72 sold only 339 boxes. How many boxes did both of the troops sell?

Both troops sold 935 boxes.

7. The boys collected 436 pounds of paper for the school paper drive. The girls collected 509 pounds. How many pounds of paper did the boys and girls collect?

They collected 935 pounds of paper.

8. John went to the store with 45 cents. He bought a ball for 33 cents. How much money did he have then?

He had 12¢ then.

9. Carol put 15 candles on Mother's cake. Mother laughed and said, "Carol, you know I am forty-three years old!" How many more candles should be on Mother's cake?

28 more candles should be on the cake.

Set 10

Solving Problems

Use your own paper. First, write the number of the problem. Next, write an equation. Last, write a sentence which tells the answer to the problem.

1. The 14 Brownies of Mrs. Lake's group invited the 18 Brownies of Mrs. Webster's group to go on a picnic with them. How many Brownies would be going to the picnic if everyone could go?
   32 Brownies would be going.

2. Sixteen boys from the Center School are in the Little League baseball team. Twenty-three boys from the West School are members. How many more boys from the West School than the Center School are members of the Little League?
   There are 7 more boys from the West School.

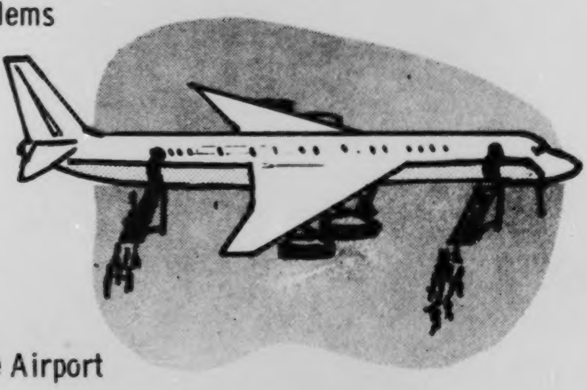
3. If 535 cartons of milk were bought by the school lunchroom on Monday, and 458 cartons were bought on Tuesday, how many were bought both days? 993 cartons were bought on both days.

4. If 415 of these cartons were chocolate milk, how many were regular milk? 578 cartons were regular milk.

5. Barbara has a collection of buttons. Penny brings her collection of 180 buttons over to Barbara's house. The girls count the buttons and find there are 321 all together. "Oh," said Barbara, "I forgot to count mine!" How many buttons did Barbara have? Barbara had 141 buttons.

Set 11

Solving Problems



A Visit to the Airport

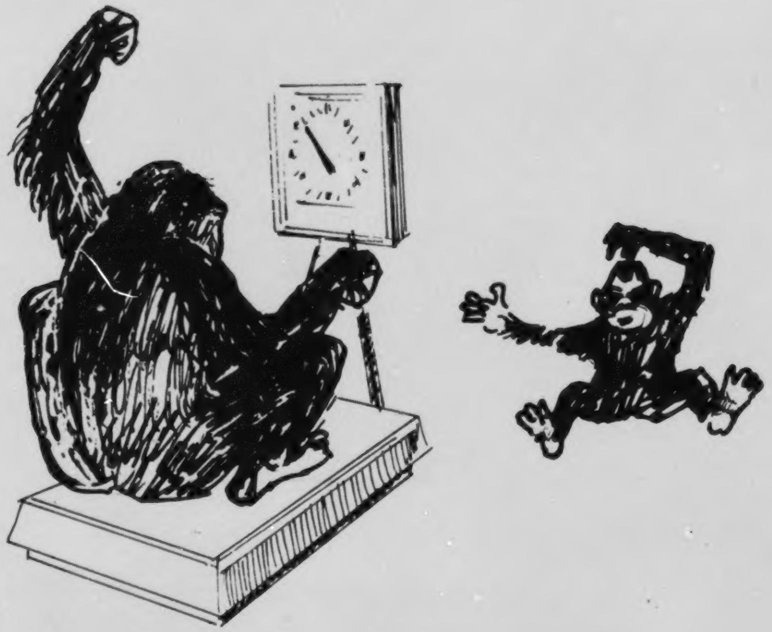
Follow the directions for Set 10.

1. Paula and John went to the airport with their parents. Paula counted 33 passengers entering at the front of a jet. John counted 82 passengers entering at the back. How many passengers did they count? They counted 115 passengers.

2. One hundred seventy-six pieces of baggage were piled on several trucks. One of these trucks pulled away with 39 pieces of baggage. How many were on the other trucks? 137 pieces were on other trucks.

3. Paula and John looked around on the observation platform and counted 37 people including themselves. If 16 of the people were children, how many adults were there? There were 21 adults.

4. A jet landed and the pilot said he had 432 more miles to go. If the total trip he makes is 906 miles, how far had he already gone? He had already gone 474 miles.



At the Zoo

Write your answers to these questions.

Betty and Jim are at the zoo. The zoo keeper wanted to weigh a baby gorilla, but the baby gorilla would not stay on the scale. The zoo keeper solved the problem by weighing the mother gorilla first. She weighed 162 pounds. Then he weighed the mother gorilla holding the baby gorilla in her arms. Together they weighed 190 pounds. How could the zoo keeper know what the baby gorilla's weight was?

Subtract 162 from 190. 190-162 = 28
The baby weighed 28 pounds.

Jim said he was going to give his bag of 75 peanuts to the gorillas. If they ate 27, how many were not eaten?

48 peanuts were not eaten.

Set 12

Solving Problems

Write an equation and an answer sentence for each problem.

1. Father had 22 tulip bulbs to plant. He found that 6 of them were not good and he threw them away. He bought a dozen more tulip bulbs at a sale. How many bulbs did he have then?

He had 28 bulbs then.

2. Jim had 15 glass marbles and 13 steel marbles. He gave 12 of his marbles to Sam. How many marbles did Jim have then?

Jim had 16 marbles then.

3. Sue had 11 pieces of dollhouse furniture and Jackie had 13 pieces of doll furniture. While they were playing together they broke 4 of the chairs. How many pieces of doll furniture did they have then?

They had 20 pieces of furniture.

4. Mother baked 24 chocolate cupcakes and 18 white cupcakes. She sent 10 of the chocolate cupcakes to Grandmother. How many cupcakes does Mother have now?

She has 32 cupcakes now.

5. Mr. Jones delivered 22 quarts of milk to houses in one block, 13 quarts of milk to houses in another block, and 9 quarts of milk to houses in the third block. How many quarts of milk did he deliver to the houses in those three blocks?

He delivered 44 quarts of milk.

6. Fifteen airplanes were at the airport. In one hour 3 airplanes took off and 6 airplanes landed. How many airplanes were at the airport then?

18 airplanes were at the airport.

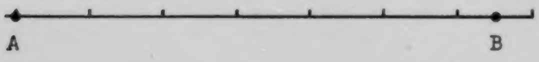
Chapter VI

LENGTH AND AREA

Background

The concept of measurement is an important one in our experience. Quantities to be measured involve such diverse things as weight, time, length, area, angle, volume, temperature, and speed. In each case the process of measurement consists in selecting a unit quantity and then counting the number of these units which match, in some sense, the quantity being measured. Thus, in measuring the length of a line segment, we select a unit segment and ask how many congruent copies of the unit segment laid end to end with no overlap are required to cover the segment being measured. Similarly, to find the area of a region one selects a unit region and counts how many congruent copies of this unit, having only edges in common, are necessary to cover the region being measured. Note that in both these cases the concept of congruence plays a major role. This chapter is devoted to extending the idea of measuring length, which has already been introduced in the previous grades, and to introducing the measurement of area.

A characteristic property of measurement is that it is not exact. Thus, if a unit segment is laid off repeatedly on a segment $\overline{AB}$, the likely situation is the following:

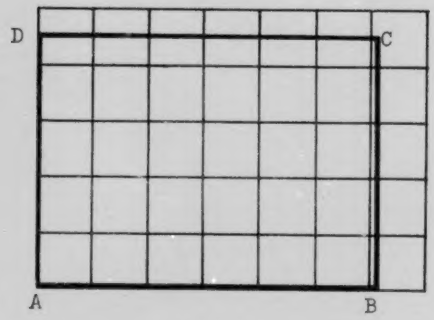


where $\overline{AB}$ is more than 6 units long but less than 7 units long. This situation is commonly treated by taking what seems to be the better of two approximations and saying this is the length to the nearest unit. Thus, here the length of $\overline{AB}$ to the nearest unit is 6 units.

One might think this difficulty could be overcome by taking smaller units. Generally a measurement can be so improved. But no matter how small the unit (or how accurate the measuring instrument) we always face the same kind of problem. That is, we have to decide which of the two adjacent unit marks represents the more acceptable as a measure of the length we are trying to find.

Use of these approximate measurements can cause certain difficulties. For example, suppose the three sides of a triangle are each 8 inches long to the nearest inch, but are actually close to 8 and a half inches. If we write $8 + 8 + 8 = 24$, we might conclude that the perimeter is 24 inches, although we know the correct value for the perimeter is over 25 inches. The problems of computing with approximate data seem too involved for this grade level. However, they are avoided in this chapter by treating all stated measurements as exact. The figures to be measured for computing perimeters have supposedly been drawn so measurements are virtually exact.

The problem of approximation for areas is even worse, since, in fitting unit square regions on a rectangular one, the following sort of situation is likely,



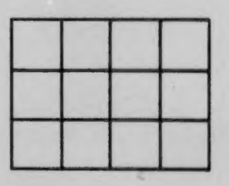
in which all we are sure of is that the area of region ABCD is between 20 units and 30 units. This again is dealt with in this chapter by the process of considering only the case of exact fitting. If the pupils ask questions about this, simply indicate that the more complicated case will be considered again in later grades.

In recording any measurement, it is necessary to give both the number and the measuring unit used. It is meaningless to say that a segment has a length of 5 or that an object has a weight of 2. Instead, we speak of a length of 5 inches or a weight of 2 pounds. If length is measured in terms of some segment which is not standard (and hence has no specific name) we may speak of a length of 5 units.

The early part of this chapter is largely concerned with finding perimeters of polygons. Let the lengths of the three sides of a triangle be 3 ft., 5 ft., and 6 ft. If congruent copies of these segments are laid end to end along a line, the length of the resulting segment is called the perimeter. It may be described intuitively as the distance travelled in tracing a curve. Here the perimeter is clearly 14 ft. It is a temptation simply to call this process addition and to say that 14 ft. is the sum of 3 ft., 5 ft., and 6 ft. However, we have always used the word addition to mean an operation on numbers only. In measurement we have used the word measure to mean the number of units. Hence we have chosen not to speak of addition of lengths, but to observe that the measure of the perimeter is the sum of the measures of the three sides. Thus the answer results from $3 + 5 + 6 = 14$, which is a statement about numbers. In indicating the perimeter, however, one must show both the measure and the unit; i.e., the perimeter is 14 ft., the measure is 14.

Please note carefully that there is no mathematical reason why the meaning of the word addition cannot be extended to cover this operation on lengths or indeed on measurements of any kind. This will almost certainly be done at some time. When it is done, however, the pupil should be made conscious of the fact that an extension is being made.

Similar remarks apply to areas. The number of unit regions covering the rectangular region below is the product of the numbers of unit segments in the two sides.



Thus, the number of unit square regions above is given by the statement $3 \times 4 = 12$ and the area would be given as 12 square units. Notice that when a unit of area is a square region each side of which is one unit of length, we customarily refer to this unit of area as a square unit. Thus the square region one inch on a side is the square inch, the square region one foot on a side is the square foot, and so on.

A major reason for discussing area, as has been done here, is that the fitting of square regions on rectangular regions gives rise to arrays of square regions. Hence, it is both a motivation and an application of the work on multiplication. Notice that much use has been made of coordinate systems, but very little is done here with standard units.

VI-1. Length of a Curve

Objectives: To review the standard units of length: inch, foot, yard.

To strengthen ability at estimations in these units.

To introduce finding the length of a curve by "straightening" a string model of it.

Vocabulary: Abbreviations for inch, foot, yard.

Materials: Foot rules (preferably marked only in inches) for pupils, yardstick, string (a piece about 15 inches long for each pupil and a ball of it to use for joint projects; be sure it does not stretch easily).

Suggested Procedure:

Take some convenient object like a pencil and ask the class to guess the length of it. (This object should be short enough so that the guesses will be given in inches.) Record the guesses and then have some child measure it and find the length (to the nearest inch). Have a one-inch segment drawn on the chalkboard. Repetitions of the exercise of estimating and verifying can be interesting and will aid in developing the perception of length.

Now do the same thing with something longer, say the edge of the chalk tray. In all probability, at least some pupils will make estimates in feet. Discuss with the class why a larger unit is more convenient here, but be sure the pupils understand that we still could use inches if we wished. Have a segment one foot long drawn on the chalkboard. Now have the measurement done to check the estimates. This may be done with several lengths. (In fact this exercise of estimating distances and then verifying would be a desirable exercise at intervals for some time.)

Now discuss with the class situations when a still longer unit might be useful. This should lead to suggestions of the yard and the mile. The yard, for example, would be feasible for measuring the school corridor, while the mile is more appropriate for measuring the distance from New York to Chicago. Have a segment one yard long drawn on the chalkboard. Some experience in estimating distances in yards may also be given. Review with the class the facts (probably already familiar) about the relations among these units, i.e., 12 congruent copies of the inch segment just cover the foot segment and 3 congruent copies of the foot cover the yard. This may be a good time also to introduce abbreviations for these units as a convenience in recording the results: 12 inches, 12 ins., 12"; 1 foot, 1 ft., 1'; 3', 1 yard, 1 yd.

Discuss with the pupils the idea that so far we have talked only about lengths of segments. Now raise the question of lengths of other curves. For example, if you have a globe in the room, ask about the length of the equator. In any case, choose some curve which is not a line segment. Be careful that the object to be measured is actually a curve. Try to elicit from the class the idea of laying a string along the curve in question and then straightening out the string to be measured. In the case of the globe, one simply draws the string snugly around the equator. Here again it would be an interesting exercise to have the class estimate the length. A second experiment of this type might be done finding the total length of the rim of a desk by wrapping a string around it and then measuring the string.

Pupil's book, pages 315-316: Distribute the pieces of string to the pupils. (Length of fine wire would be as good or better.) The pupils will need guidance in how to hold the string so they have the correct piece of it to measure. Notice that this procedure is quite crude. The important point here is to stress the idea of the length of a curve, not to develop any particular proficiency. Explain that you are only looking for answers to the nearest inch. After doing part A together, parts B, C, and D could be done by different groups of children in the class.

Pupil's book, pages 317-318: Problems are simply to review the relations among the different units of length. They may be used either together or working independently.

Further activities and enrichment:

1. In Chapter III a suggested activity was using the scale of miles on a map to find the distance between cities. It was noted that the distances there obtained were airline distances. Using the method of this section, one could lay a string along the curve showing the road from one city to another. This string could now be measured using the scale of miles to find the road distance between two cities.
2. A bright pupil might be interested in using the scale of miles on the globe to try to find the actual length of the equator on the earth. Note, however, that the numbers in question get rather large.
3. (a) A good problem is to find the shortest distance from Los Angeles to New York. (This is done by drawing a string snug on a globe between the points marked with these names and then converting the distance to miles, using the globe's own scale.) Some child might wish to check this with the distance given in some airline map, which might give a distance somewhat shorter than a highway distance chart.

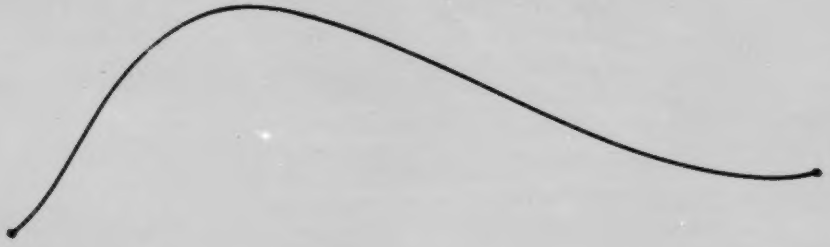
  (b) From observations available in (a) above, it might be easy to stimulate answers about the best stop on a one-stop flight between the two cities. Suggested stops to compare might be Minneapolis, Denver, Houston. (Which of these cities lies nearest the "shortest distance" route?)

4. Some historic "around the world" flights could be studied to see if they really went around the world in the sense of nearly following a great circle route, or if they went only around the North Pole.

5. Some child may wish to talk about trajectories of some satellites with nearly circular orbits.

Lengths of Curves

1.



Look at this picture of a curve.

Place your string along the curve.

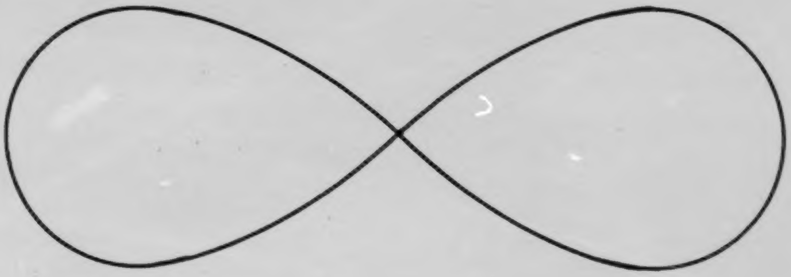
Measure the part of the string you used.

The length is 6 inches.

This is called the length of the curve.

You have used a string to find the length of a curve.

2.



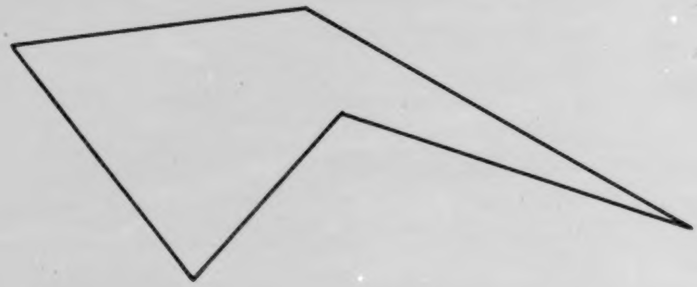
Look at this picture of a curve.

Use your string to measure it.

The length of this curve is 14 inches.

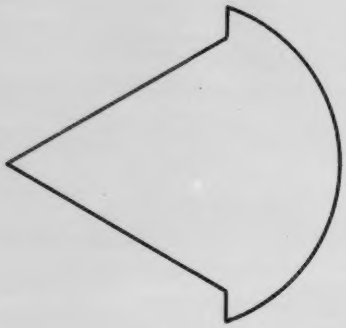
Lengths of Curves

3. Use your string to find the length of the curve drawn below.



The length of the curve is 11 inches.

4. Is the length of the curve below greater or less than 10 inches? less



How much less than 10 inches is it? 4 inches

Length

1. The seat of a chair is 15 inches from the floor. This is

   3 inches more than a foot.

2. The length of the edge of a desk is 22 inches. This is

   2 inches less than 2 feet.

3. Henry's seat is 7 feet from the door, while Harold's is 3 yards from the door. Which one is farther from the door?

   Harold

   How much farther? 2 feet.

4. To reach the drinking fountain Jane has to go 2 feet more than 4 yards. How many feet is this? 14 feet

5. Neil finds he is 14 feet from the chalkboard. Is this more or less than 6 yards? 4 feet less than 6 yards.

Length

6. In playing Pin the Tail on the Donkey, Judy places the tail 10 inches from the correct spot. For Alice, the distance is 4 inches less than a foot. Who was the winner? Alice

7. Henry and James use a yardstick to find the height of a room. They find it is 2 feet more than 2 yards. The room is 8 feet high.

8. One space capsule was 5 feet, 8 inches high inside when in flight. The astronaut in his suit was 6 feet 5 inches tall. Could the astronaut stand up straight in the capsule? No.

Why? Six feet five inches is taller than five feet eight inches.

9. Amy's doll buggy is 3 feet, 5 inches long. Jerry's truck has a cargo body 1 foot, 9 inches long. Will Amy's buggy fit into the cargo body of Jerry's truck? No. Why?

Phrasing of answers will vary.

VI-2. Perimeters of Polygons

Objectives: To introduce perimeter as the length of a polygon.

To determine perimeter by laying off segments on a straightedge.

To determine perimeter by adding measures of the component segments.

Vocabulary: Perimeter

Materials: Yardstick (for teacher), rulers (for pupils), bulletin board, thumbtacks, string.

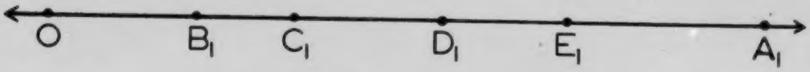
Suggested Procedure:

Remind the class of the work in the last lesson on finding lengths of curves. Ask if this method of laying string on a curve was very precise. (The answer should be no. It is quite crude and there may very likely have been arguments over the correct answers in the last lesson.) Indicate that this time we will look for better ways of finding the lengths of polygons. Review with the class what a polygon is. (A simple closed curve which is a union of line segments.) Ask whether any of the curves measured in the last lesson were polygons. (Yes, 3 was a polygon.)

Indicate that a special word is used for the length of a polygon. It is called the perimeter of the polygon. In the last lesson we found the perimeter of the curve in 3 to be about 11 inches.

Pupil's book, page 319: Have on the board a drawing of a line at least a yard long with a point 0 marked near one end. Ask a pupil to use either his string or the edge of a sheet of paper to mark on the board, starting at 0, a segment which is congruent to $\overline{AB}$. Let him mark the endpoint B_1

to remaind us it came from B. Now have a second pupil lay off a segment starting at B_1 and congruent to $\overline{BC}$, marking the endpoint C_1. Continue until all the segments have been marked, yielding a figure similar to the one below.



Lead the class to see that they now have congruent copies of the segments of the polygon laid end to end. Try also to elicit the idea that this is exactly what we get if we lay a string on the polygon very carefully and straighten it out.

It is easier to do this for a polygon than for a figure with a curved boundary. One way is: (1) trace the figure on tracing paper, marking each vertex (corner) clearly with a dot; (2) post tracing paper smoothly on a bulletin board designed to accept thumbtacks; (3) place a thumb tack accurately and firmly at each corner, leaving enough space to allow the string to slip under the head; (4) put a small pencil mark on the string near its starting end; (5) run the string around outside the tack at A, outside at B, inside at C, and outside at D, E, and A; (6) pull the string snug but not tight. Now the string should lie almost exactly so as to cover all sides of the polygon. The string should cover itself along a part of the segment $\overline{CA}$. The original pencil mark should be here. All that is needed now is to mark the unmarked string at the place where it passes that mark. Then, unwind the string. Make sure the marks are preserved. If all the work has been done accurately, the measures between the marks on the string and the points 0 and A_1 on the line should be (approximately) equal.

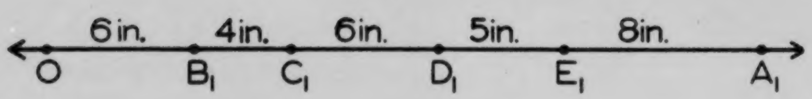
This should be verified directly by placing the string against the line. If there is a slight difference, it will help to emphasize that all measurement (and especially the crude kind we are doing here) is inherently inaccurate.

It may also be desirable to have two different (sets of)children measure the line segment $\overline{OA_1}$ and the length of string between the marks in the same units, say inches, and then to compare their results. Further small discrepancies are likely to occur. These can be used both to remind the children to be careful in their use of measuring implements and to illustrate further the inherent inaccuracies involved in the process of measurement.

There are other variations such as actually measuring the difference between the length of string as marked and the length of the segment $\overline{OA_1}$. This measurement should be taken while the string is held against the segment on the chalkboard. This measurement might be compared with the difference of the measurements obtained from the segment and the string separately. (The perimeter of the figure as it appears in the pupil's book is 29 inches.)

Leave this material on the board, but turn with the pupils to page 320 in the pupil's book. Have the pupils follow the instructions for finding the perimeter using the same procedure as above.

Now have the pupils turn back to page 319 in the pupil's book. Observe that the figure consists of segments so that we could, if we wished, measure each of these segments. Have some child measure $\overline{AB}$. Elicit the fact that this is also the length of $\overline{OB_1}$. Why? (Because $\overline{AB}$ is congruent to $\overline{OB_1}$.) Record this length above $\overline{OB_1}$. Treat the other segments similarly. The drawing on the chalkboard will then look like this:



Remind the pupils that before the perimeter was found just by measuring $\overline{OA_1}$. Try to elicit the idea that another way to find the number of inches in the perimeter would be to add the numbers of inches in each segment. An equation that describes this would be

$$6 + 4 + 6 + 5 + 8 = 29.$$

This equation should be written on the board. Thus, the perimeter is 29 inches. This also should be written on the board. Notice the convention at this point that we add only numbers, not inches. Be very sure, however, that the pupils do not simply say the "answer" is 29. To describe a perimeter (or any length) one must give both the number and the unit involved, as was done above in indicating the perimeter.

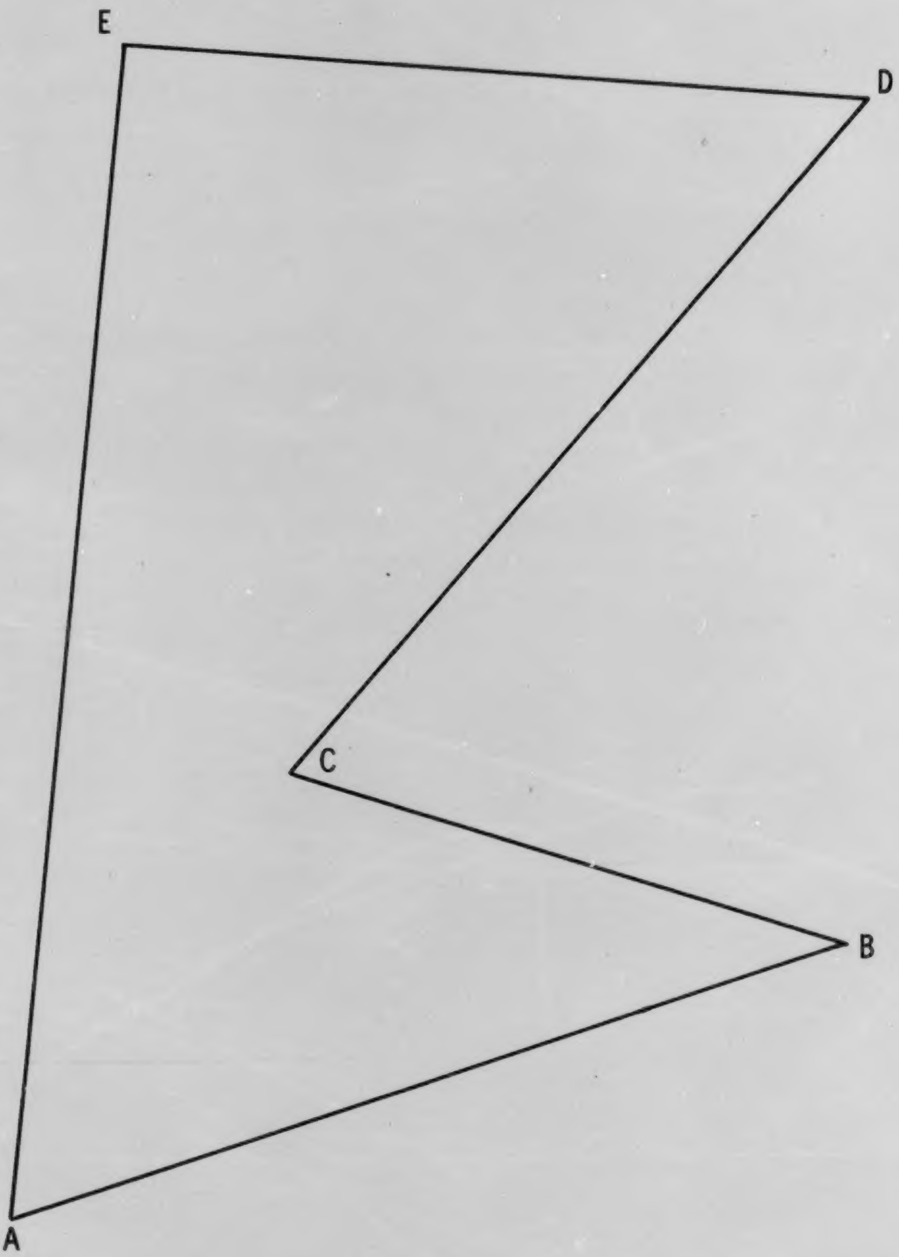
Pupil's book, page 321: Go through this page together. Notice in particular the questions to be sure that the student understands clearly that the symbols 40 feet and 14 feet refer to the playground, not to the picture. Some children find this confusing. Be sure that in the last two lines the pupils write 108 feet, not just 108.

Pupil's book, pages 322-323: This problem seeks to check again on the understanding that a length must involve both number and unit.

Pupil's book, pages 324-325: These pages review the ideas of the section.

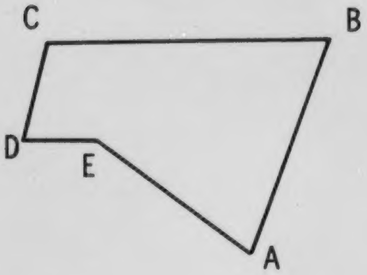
Perimeters

1. Find the perimeter. 29 inches.

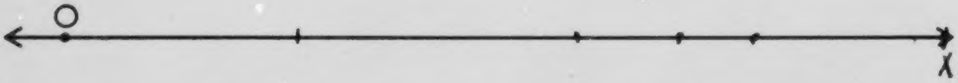


Perimeters

2. We will find the perimeter of the polygon drawn below.



Mark a segment congruent to $\overline{AB}$ on the line below, starting at O.



Next to it mark a congruent copy of $\overline{BC}$.

Make congruent copies of all the segments.

Call the last endpoint X.

Measure $\overline{OX}$.

The length of $\overline{OX}$ is 6 inches.

The measure of $\overline{OX}$ is 6.

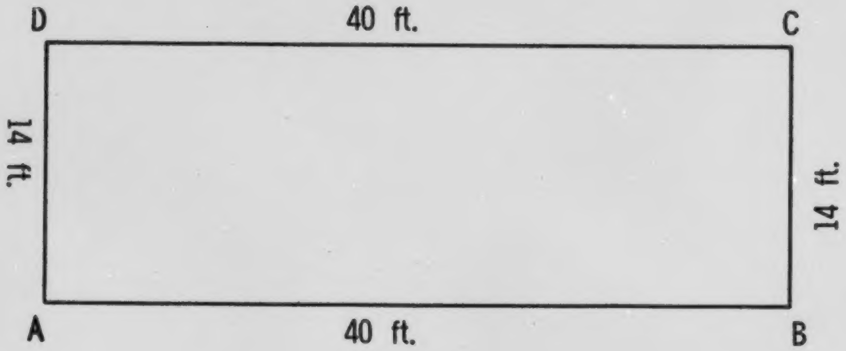
The perimeter of the polygon is 6 inches.

Perimeters

3. This is a picture of a playground.

We want to know how much fence is needed for the playground.

That means we want the perimeter of the playground.



Below $\overline{AB}$ is a mark like this: 40 ft.

Does this mean that $\overline{AB}$ is 40 ft. long? No.

Write the mathematical sentence which we have to use.

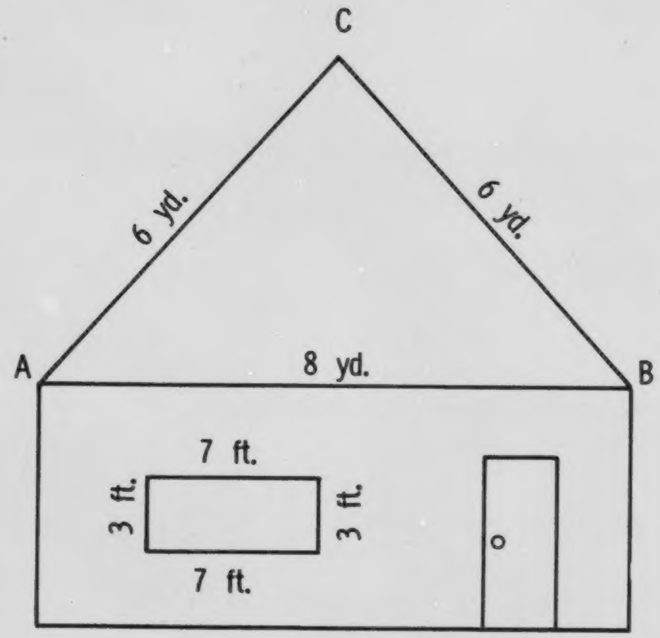
$40 + 14 + 40 + 14 = 108$

The perimeter of the playground is 108 feet. .

The length of fence needed is 108 feet .

Perimeters

4. The Jones family decided to decorate the front of their home for the Christmas season.



Johnny wanted to put a string of colored lights on the house along the triangle ABC.

Mary wanted to put a string of colored lights around the window.

Mr. Jones said he would buy lights for the window or the roof. He would *not* buy lights for both. He would decorate the one which required the shorter string of lights.

4. Johnny measured the three sides of the triangle.

He added the numbers in his measurements.

He wrote the equation

$$6 + 6 + 8 = 20.$$

Mary measured the four sides of the window.

She added the numbers in her measurements.

She wrote the equation

$$3 + 7 + 3 + 7 = 20.$$

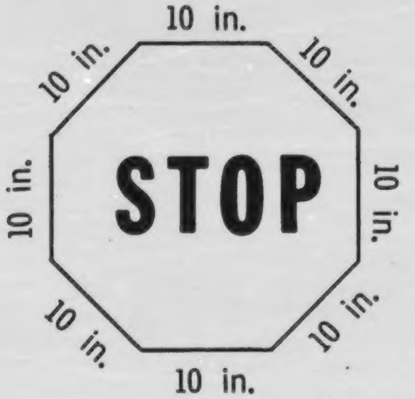
The two numbers were the same.

What other fact did Mr. Jones have to know? The unit of measure.

Did the Jones family decorate the window or the roof? The window.

Review

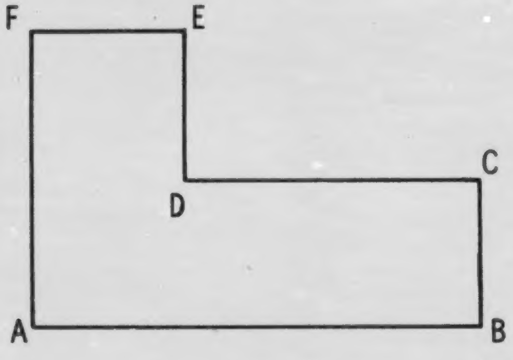
1. This is a picture of a STOP sign. A thin black border is to be painted around the edge.


10 in.
10 in.
10 in.
10 in.
STOP
10 in.
10 in.
10 in.
10 in.
10 in.

How many inches of border must be painted? 80 inches.

The edge of the sign is a polygon.
The perimeter of the polygon is 80 inches.
The measure of the perimeter of the polygon is 80.

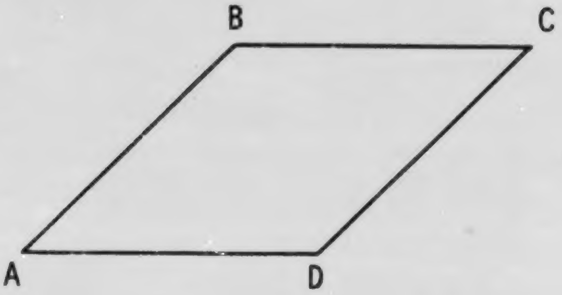
2. Use your ruler to find to the nearest inch the perimeter of the polygon below.



The perimeter of ABCDEF is 10 inches.

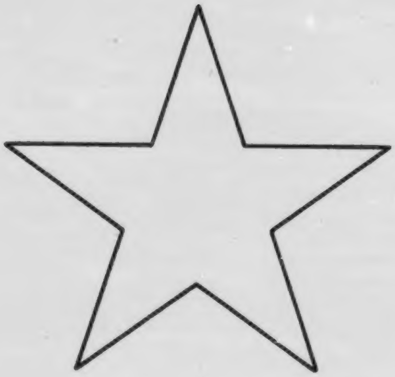
Review

3. Use your ruler to find to the nearest inch the perimeters of the polygons below.



The perimeter of ABCD is 8 inches.

The measure of ABCD is 8.



The perimeter of the star is 10 inches.

VI-3. Using Different Units

Objective: To deal with lengths expressed in more than one unit, such as feet and inches.

Vocabulary: (No new words.)

Materials: Colored chalk

Suggested Procedure:

Have a chart like the one below drawn on the chalkboard.

1 ft	
	24 in.
3 ft.	
4 ft.	
5 ft.	
6 ft.	

Remind the children that a length may be given in several ways. Refer to the chart and ask for another name for 1 foot. (12 inches.) Proceed to fill in the rest of the chart. This may require some prompting. For example, once it is established that there are 36 inches in 3 feet, you can elicit the fact that in 4 feet the number of inches must be 12 more. Since 36 + 12 = 48, this means there must be 48 inches in 4 feet. Since these facts will not be familiar to all the class, the chart may be left on the chalkboard to use during the discussion.

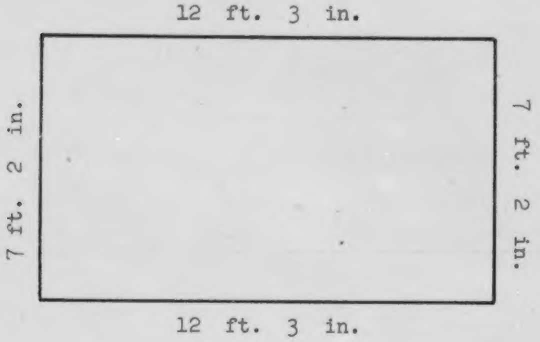
Now ask for another way of saying 46 inches. By referring to the chart, notice that it is more than 3 feet, but less than 4 feet. It is 10 inches more than 3 feet. This is commonly written as 3 feet 10 inches.

Discuss with the pupils the situations they can recall where distance is expressed in this way. Examples might be:

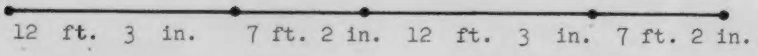
A man 6 feet 3 inches tall.
A pole vault of 16 feet 8 inches.
A broad jump of 23 feet 5 inches.
The edge of a table 4 feet 2 inches long.

This is a very common way of expressing lengths. To emphasize the equivalence of different names, have the pupils turn to 1, Pupil's book, page 326 and complete these tables, referring to the chart on the board as needed.

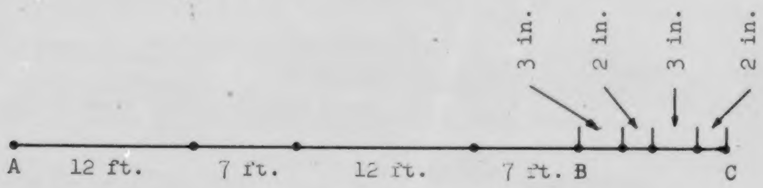
Now draw on the chalkboard a figure like the one below, including the length markings.



Indicate that this is a picture of the floor of a room and that you are interested in the perimeter. Ask the class to imagine laying a string around the floor and straightening it out (as we did in Section 1). The string would then look something like this:



Make such a drawing on the board. Have the class imagine that they take a pair of scissors and cut this string into eight pieces. For example, the first section would be cut into a 12 foot piece and a 3 inch piece. Colored chalk might be used to show the longer and shorter segments in contrasting colors. Then imagine rearranging the pieces as shown below. Draw this on the board, again using the two colors.



The line segment to the right of B represents a length of (3 + 2 + 3 + 2) inches.

Elicit the fact that this has not altered the length. That is, the desired perimeter is the length of $\overline{AC}$. But we know that the number of feet in $\overline{AB}$ is given by the equation

$$12 + 7 + 12 + 7 = 38,$$

and the number of inches in $\overline{BC}$ is given by the equation

$$3 + 2 + 3 + 2 = 10.$$

Thus, the length of $\overline{AC}$ may be written as 38 feet 10 inches. This is the perimeter of the floor.

Lead the class from this example to the understanding that in combining lengths, one adds the numbers of feet and the numbers of inches separately.

Pupil's book 326-327: Question 2 is routine but in 3 the perimeter comes out as 23 feet 26 inches. The last few questions in 3 try to lead the pupils to rewrite this as 25 feet 2 inches. This may call for discussion. Question 4 involves the same ideas applied to yards and feet. It is hoped that the pupil will see that, while 12 yards 5 feet is correct, a better way of writing the answer is 13 yards 2 feet.

Pupil's book, pages 328-329: Review.
The problems may be done by pupils working independently or together, as you see fit.

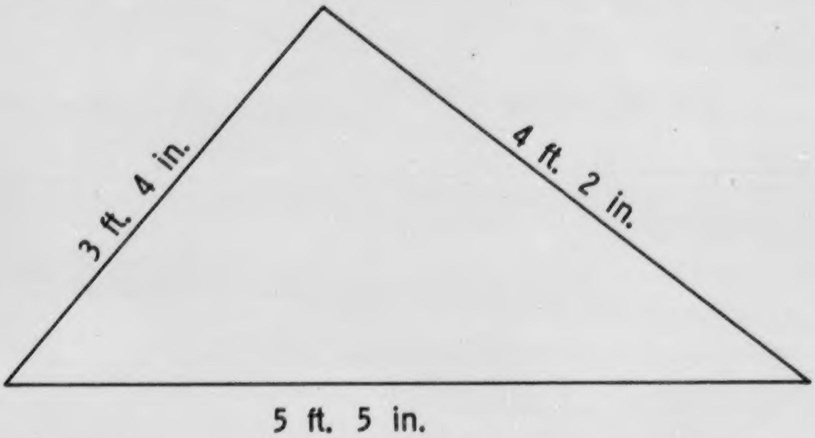
Using Several Units for Perimeters

1. Complete the table. Give another name for each length.

6 ft. 6 in.	78 inches
3 ft. 7 in.	43 in.
1 ft. 9 in.	21 in.
2 ft. 10 in.	34 in.
5 ft. 7 in.	67 in.
4 ft. 7 in.	55 in.

2. This is a picture of a triangle.

We shall find its perimeter.



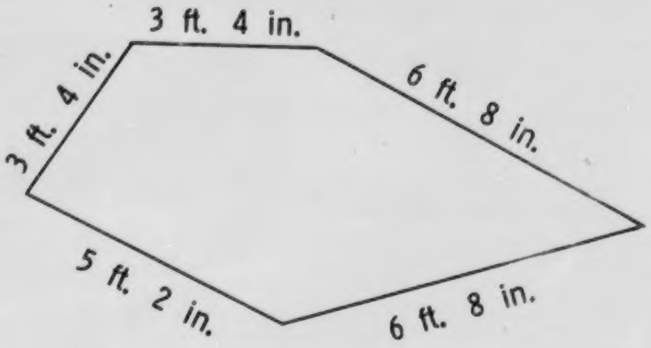
The equation for the number of feet is 3 + 4 + 5 = 12.

The equation for the number of inches is 4 + 2 + 5 = 11.

The perimeter is 12 ft. 11 in..

Using Several Units for Perimeters

3. This is a picture of a polygon.



How many sides does it have? 5

We shall find its perimeter.

The equation for the number of feet is 3 + 3 + 6 + 6 + 5 = 23 .

The equation for the number of inches is 4 + 4 + 8 + 8 + 2 = 26 .

The perimeter is 25 feet 2 inches.

Is the length shown in inches more than a foot? Yes

The length in inches is the same as 2 feet 2 inches.

4. The lengths of the three sides of a triangle are

4 yd. 2 ft., 5 yd. 1 ft., and 3 yd. 2 ft.

The perimeter in yards and feet is 13 yd. 2 ft. .

Review

1. The four sides of a flower bed have lengths 21 ft. 4 in., 10 ft. 6 in., 21 ft. 4 in., and 10 ft. 6 in.

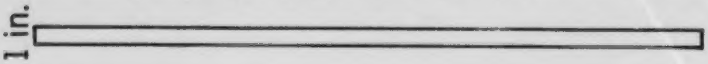
   A fence is built around it.

   The length of the fence is 62 ft. 20 in. .

   Is the length shown in inches more than a foot? Yes.

   If it is, write the length in a different way. 63 ft. 8 in.

2. This is a picture of a yardstick. It is one inch wide.

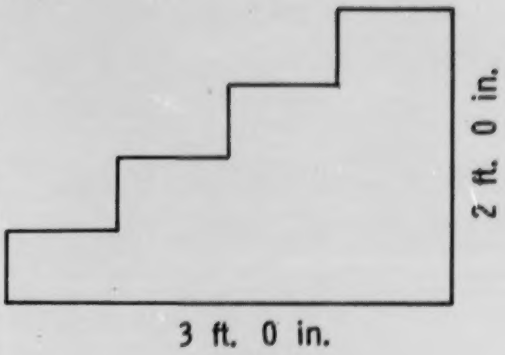


   Its perimeter in yards and inches is 2 yd. 2 in. .

   Its perimeter in feet and inches is 6 ft. 2 in. .

3. This is a picture of a polygon.

   It looks like steps.

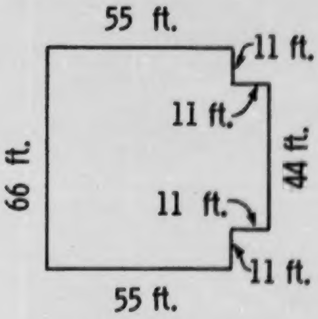

2 ft. 0 in.
3 ft. 0 in.

   Each step is 6 inches high.

   Each step is 9 inches wide.

   The perimeter of this polygon is 10 ft.

4. This might be a picture of a launching pad for an interplanetary missile.


55 ft.
11 ft.
11 ft.
66 ft.
44 ft.
11 ft.
11 ft.
55 ft.

   What is the perimeter of the pad? 264 ft.

VI-4. Using Fractional Units

Objective: To use the half and quarter inch scales on the ruler.

Vocabulary: (No new words.)

Materials: Rulers (preferably marked only in half- and quarter-inches.)

Suggested Procedure:

Discuss with the class the fact that so far in this chapter the smallest unit used has been the inch. Ask if there are times when we might want to use a smaller unit. Bring out the fact that for short lengths we often want a more careful description of a length than to the nearest inch. The diameter of a ring to go on your finger is one illustration.

Have the class examine their rulers, and notice that the half inch marks divide each inch segment into two congruent parts. Similarly, the quarter inch marks divide each half inch segment into two congruent parts. If we wish to do so, we could use either the half-inch or quarter-inch as a unit segment for measuring. Pupil's book, pages 330-331.

Looking at problems 1 and 2 discuss with the class that these lengths could be written in several ways. For example, in 2 the length of $\overline{ST}$ is given as 7 quarter inches. Have the pupils look at the ruler and count 7 quarter inches from the 0 point. Have them notice that the first four quarter inches exactly cover the one-inch segment. Thus the length of $\overline{ST}$ could equally well have been described as 1 in. 3 quarter-ins. We use the abbreviations in order to make the notation as compact as possible. Similarly, the length of $\overline{RS}$ can be written as 10 quarter-ins., or as 2 ins. 2 quarter-ins., and the length of $\overline{RT}$ as 5 quarter-ins. or as 1 in. 1 quarter-in. In the case of $\overline{RS}$, still another way of writing it would be 2 ins. 1 half-in., since 2 quarter-inches just cover one half inch. A similar discussion can be applied to the lengths listed in 1.

[Note to teacher: A length such as 4 ins. 1 half-in. is commonly called "four and a half inches". Similarly 5 ins. 3 quarter-ins. is commonly called "five and three-quarters inches". There is no objection at all to using the standard oral terminology. Until more work has been done with rational numbers, however, it has seemed better to avoid such numbers as $5\frac{3}{4}$.]

Discuss with the class that the work they did in the last lesson of adding separately numbers of feet and numbers of inches applies here to combining lengths written, for example, in inches and quarter-inches.

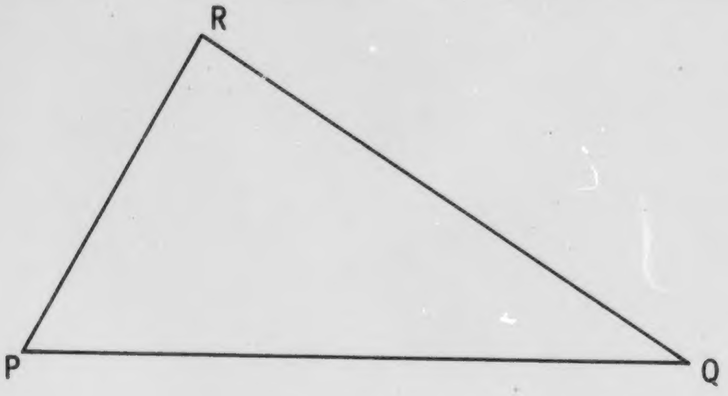
Pupil's book, page 332:

In questions 3 and 4 have the pupils write their answers so the half- or quarter-inches shown are less than an inch. Thus for 3 an answer of 34 ins. 2 half-ins. is correct, but should be rewritten as 35 ins. 0 half-in. or just 35 ins. Similarly, for 4 the answer, 19 inches 6 quarter-inches, should be rewritten as 20 inches 2 quarter-inches.

Pupil's book, pages 333-334: The review excercise should be done independently.

Measuring with Fractional Units

1.



Take your ruler.

Measure each side of $\triangle PQR$ in half inches.

Length of $\overline{PQ}$ is 9 half-inches.

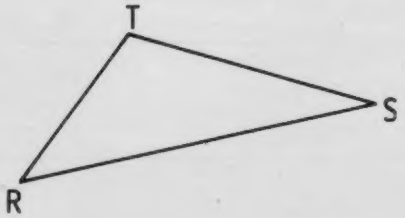
Length of $\overline{QR}$ is 8 half-inches.

Length of $\overline{PR}$ is 5 half-inches.

The perimeter of $\triangle PQR$ is 22 half-inches.

Measuring with Fractional Units

2.



Take your ruler.

Measure each side of $\triangle$RST in quarter inches.

Length of $\overline{RS}$ is 10 quarter-inches.

Length of $\overline{ST}$ is 7 quarter-inches.

Length of $\overline{RT}$ is 5 quarter-inches.

The perimeter of $\triangle$RST is 22 quarter-inches.

3. The sides of a triangle are

12 ins. 1 half-in.

13 ins. 0 half-in.

9 ins. 1 half-in.

The perimeter is 35 in. 0 half-in.

4. The sides of a triangle are

5 ins. 3 quarter-ins.

8 ins. 2 quarter-ins.

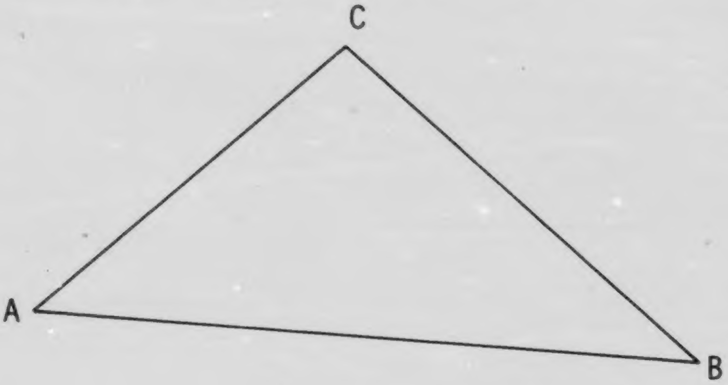
6 ins. 1 quarter-in.

The perimeter is 20 ins. $\frac{2}{4}$ in. .

Review

1. Measure the sides of △ABC with your ruler.

Show the results below.



Length of $\overline{AB}$ is 4 ins. 2 quarter-ins.

Length of $\overline{BC}$ is 3 ins. 1 quarter-ins.

Length of $\overline{AC}$ is 2 ins. 3 quarter-ins.

The perimeter of △ABC is 9 ins. 6 quarter-ins.

Do the quarter inches make more than an inch? Yes

If so, write the perimeter in a different way.

10 in. 2 quarter-in.

Review

2. Sally is giving a party.

She wants to make a sash of ribbon.

It will take 27 ins. 3 quarter-ins.

She also wants to make a bow of ribbon.

This will take 15 ins. 3 quarter-ins.

She has 43 ins. of ribbon. Is this enough? No.

3. A rectangle has sides whose lengths are

3 ins. 1 half-in., 2 ins. 0 half-in.,

3 ins. 1 half-in., 2 ins. 0 half-in.

Is the perimeter as much as a foot? No.

The perimeter is 11 in..

4. The edges of the cover of your pupil's book form a rectangle.

The sides have lengths, to nearest half-inch:

(bottom) 8 ins., 1 half-in.

(right side) 11 ins., 0 half-in.

(top) 8 ins., 1 half-in.

(left side) 11 ins., 0 half-in.

The perimeter is 38 ins., 2 half-ins. or 39 in.

The perimeter is more than than 3 feet.

The perimeter is less than than 4 feet.

VI-5. Introduction to Area

Objectives: To introduce area of a region as the number of congruent copies of some unit region necessary for covering that region.

To introduce the square region as a desirable unit.

To consider the area of a rectangular region.

Vocabulary: Area, square inch, square foot.

Materials: A rectangular region 18" × 30" drawn on the chalkboard or on paper, a supply of 6" × 6" square regions cut from paper (at least 15 of them).

Suggested Procedure:

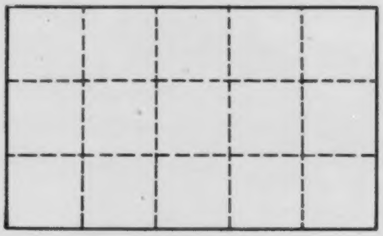
Review with the children what it means to measure a segment $\overline{AB}$. Try to elicit from them that it means selecting a unit segment and then finding how many non-overlapping congruent copies of this unit segment are necessary to cover $\overline{AB}$.

Now ask what it would mean to measure a region like, for example, the rectangular region you have drawn. Try to have the children reach the conclusion that it would mean selecting some unit region and finding how many congruent copies of this region are necessary to cover the region to be measured.

Then raise the question as to what would be a good choice for a unit region. The children will probably immediately want to choose a square region, but do not be in too much of a hurry to do this. Try to get the children to think why they prefer a square region. One reason is that they have seen in Chapter III how the plane can be covered with non-overlapping square regions.

To be sure they do not think that the square is the only region with this property, have them turn to pages 335 and 336 in the pupil's book and find the two areas where the unit region is not a square one.

After considering the possibility of other choices for a unit region, return to the square region and agree that this is the common choice of unit. Bring out the 6" × 6" square regions, suggest that one of these be used for a unit region, and find the area of the large rectangular region on the board. Have the pupils fasten the unit square regions on this rectangular region. The figure will then look roughly as follows.



Try to elicit from the class ways of finding this area. One way, of course, is just to count the unit squares. Have the pupils notice that this is exactly what they had to do in problems 1 and 2. But try to have the pupils notice that there is an array with 3 rows, each row having 5 unit regions. Thus the number of square regions must be 3×5, the measure is 15, and hence the area is 3×5 units or 15 units.

Still looking at the rectangular region on the board, ask if the longer side of the rectangle is 5 units long. This question should produce an argument. The answer depends on what we are using for a unit of length. If we use a side of our square region as a unit of length then it is true that the lengths of the sides of the rectangular region are 5 and 3 units. Otherwise this is not true. This suggests that it might be very useful if we not only choose square regions for units of area but choose them so their edges are one unit long. If lengths are being measured in inches, we will commonly use for a unit of area a square region one inch on a side. This is called the square inch. Draw a picture of this region on the board. Similarly, if lengths are measured in feet, we may use the square foot as a unit. Draw a picture of this also on the board. Do not make any particular attempt to develop the numerical relation between square inch and square foot, but notice that it will take a large number of square inches to fill a square foot. For each unit of length there is a corresponding square unit of area. To emphasize that we are making this choice, we often refer to our unit of area as a square unit.

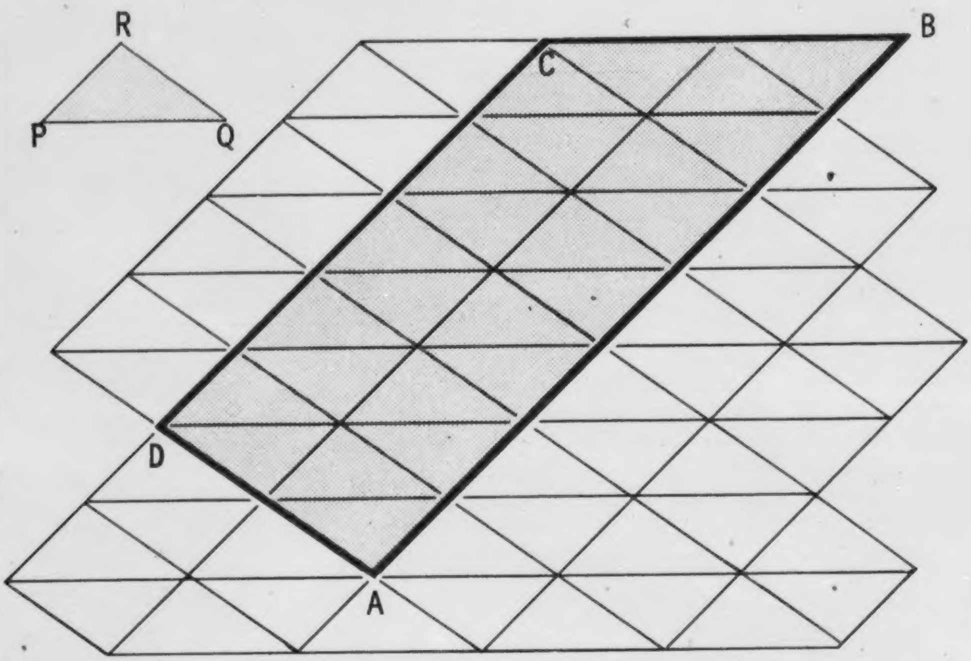
Now suggest that instead of measuring a region by fitting unit square regions on it, it might be better to fit the region onto a coordinate system, such as we used in Chapter III. Then have the children turn to page 338 in the pupil's book. For these experiences try to lead the pupils to the understanding that, for a rectangle, the number of square units in the area equals the product of the numbers of linear units in the sides (provided we have elected to use corresponding linear and square units). Call the pupils' attention to the fact that it is convenient to use bold letters as names for regions.

Pupil's book, pages 340-343:

The problems in the review pages permit some use of the principle above by having the pupils think of regions as cut into parts and then adding the numbers of units for the different parts. The problems can also be done easily just by counting unit square regions. In either case they emphasize the meaning of area. A discussion of how different pupils solved problems could be useful.

Areas

1. Let the region bounded by △PQR be the unit region.



The regions in the drawing are congruent to region PQR.

Look at quadrilateral ABCD.

It is the edge of a region.

Count the number of unit triangular regions in region ABCD.

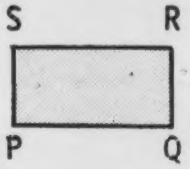
This area of the region is 24 units.

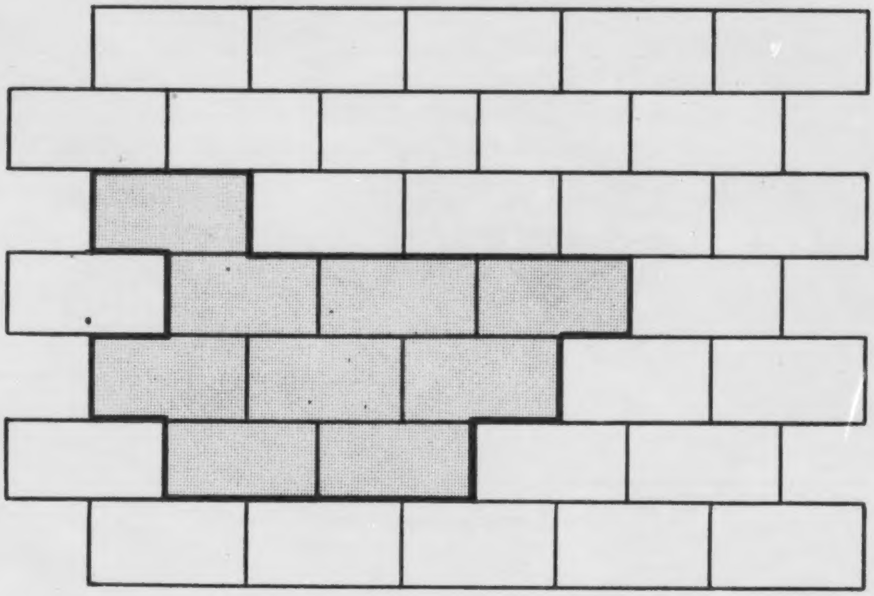
The measure of the region is 24.

Areas

2. Let region PQRS be the unit region.

The regions in the drawing are congruent to region PQRS.





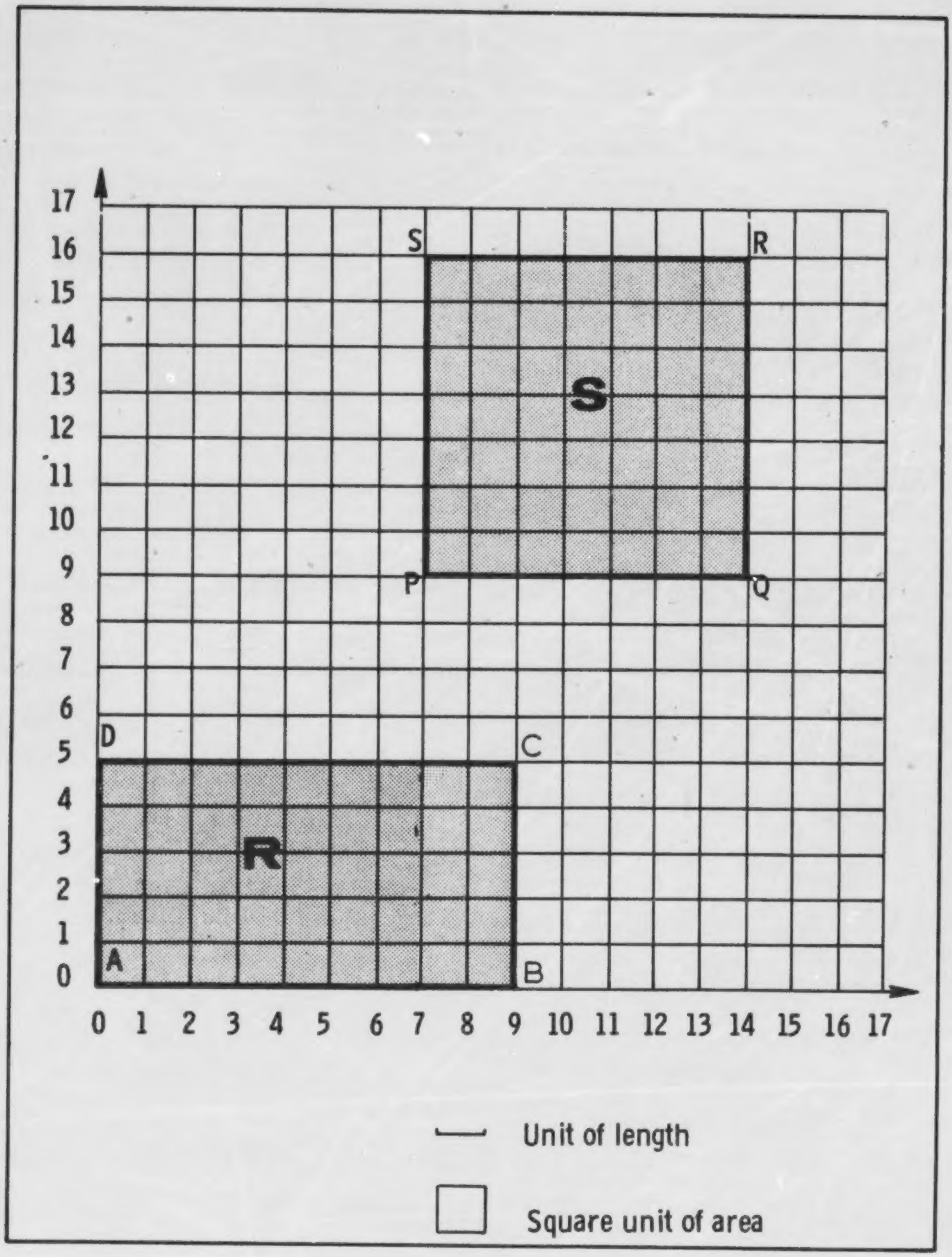
Look at the simple closed curve shown with heavy lines.

It is the edge of a region.

The area of this region is 9 units.

The measure of this region is 9.

Areas


A
B
C
D
R
P
Q
R
S
S
Unit of length
Square unit of area

Area

Look at the facing page.

See the units of length and area.

3. Look at rectangle ABCD.

It is the edge of a region called **R**.

The length of $\overline{AB}$ is 9 units.

R has 9 unit regions in each row.

The length of $\overline{AD}$ is 5 units.

R has 5 rows of unit regions.

An equation telling the number of square units in **R** is $5 \times 9 = 45$.

The area of **R** is 45 square units.

4. Look at rectangle PQRS.

It is the edge of a region called **S**.

The length of $\overline{PQ}$ is 7 units.

Could you use the numbers describing P and Q to find this? Yes

The length of $\overline{PS}$ is 7 units.

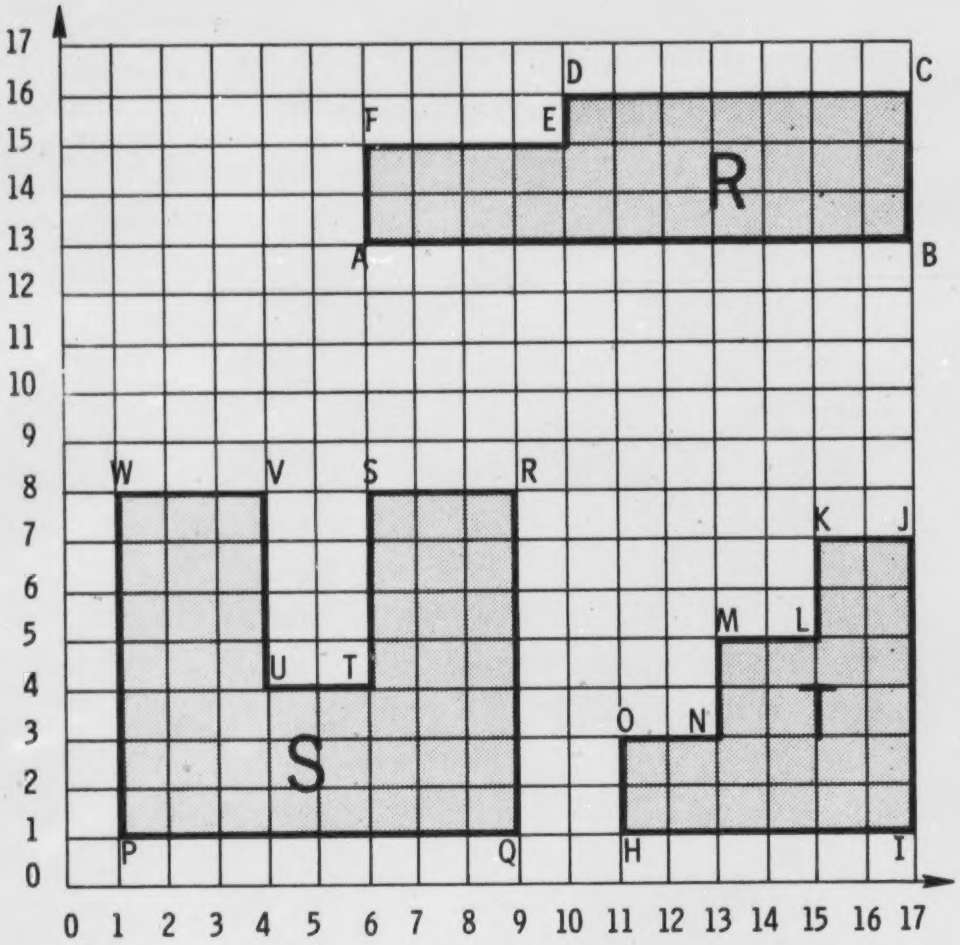
Could you use the numbers describing P and S to find this? Yes

S has 7 unit regions in each row.

An equation telling the number of square units in **S** is $7 \times 7 = 49$

The area of **S** is 49 square units.

Area


17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
D
C
F
E
R
A
B
W
V
S
R
K
J
M
L
U
T
O
N
T
S
P
Q
H
I

— unit of length

☐ square unit of area

Area

Look at the facing page.

1. ABCDEF is a simple closed curve.
   It is the edge of a region called **R**.
   The area of **R** is 29 square units.

2. PQRSTUVW is a simple closed curve.
   It is the edge of a region called **S**.
   The area of **S** is 48 square units.

3. HIJKLMNO is a simple closed curve.
   It is the edge of a region called **T**.
   The area of **T** is 24 square units.

Area

1. Look at the facing page.

The area of region ABCD is 80 square units.

Join the following points in order:

(6, 5) (9, 5) (9, 7) (6, 7) (6, 5)

Call the figure PQRS.

The area of region PQRS is 6 square units.

The area of region ABCD which is outside region PQRS is 74 square units.

2. Look at the facing page.

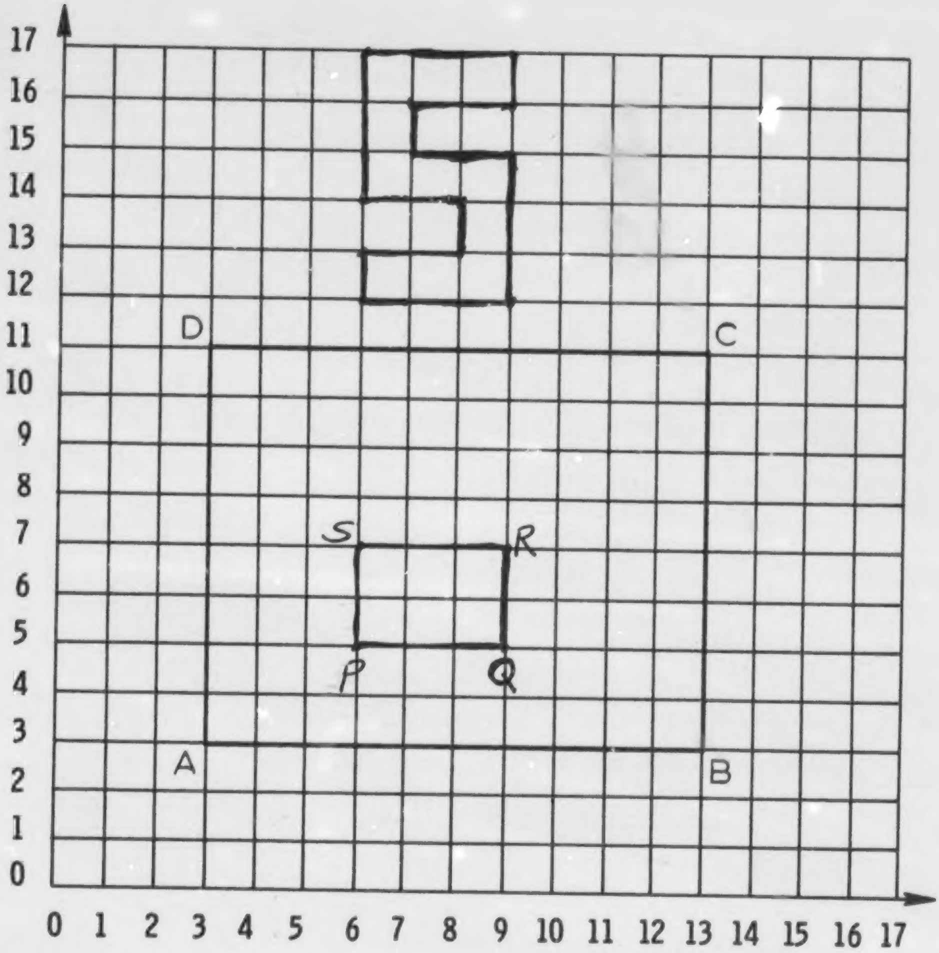
Join the following points in order:

(9, 12), (9, 15), (7, 15), (7, 16), (9, 16), (9, 17), (6, 17),
(6, 14), (8, 14), (8, 13), (6, 13), (6, 12), (9, 12).

What symbol does the resulting closed figure look like? S

What is the area of that closed figure? 11 square units.

Area


17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
D
C
S
R
P
Q
A
B

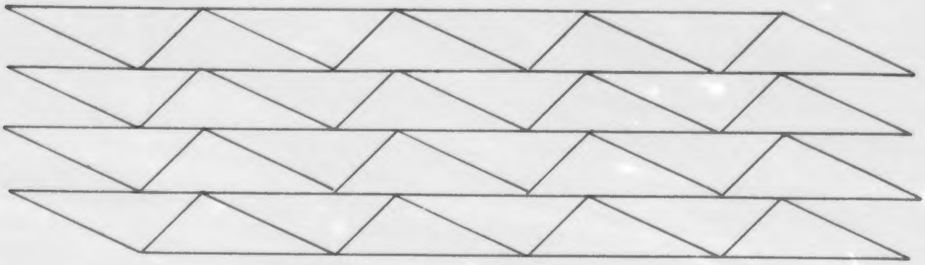
— Unit of length

□ Square unit of area

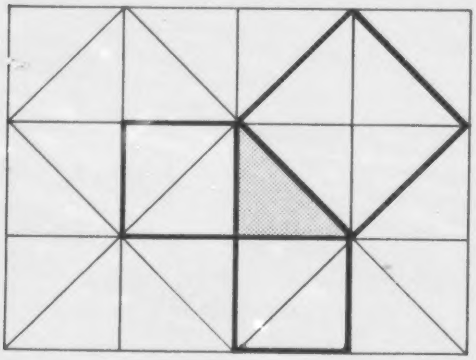
Further activities and enrichment:

1. Some pupils might enjoy working together to make a drawing of a square foot divided into square inches. They could then find the number of square inches in a square foot either by counting them directly or by breaking up the array into parts they know--for example, into four 6×6 arrays.
2. If the tiling patterns on pages 335 and 336 in the pupil's book interested some pupils, they might enjoy trying to make such patterns of their own with various kinds of congruent regions. For example, any set of congruent quadrilaterals can be fitted together.

   A different pattern, using a general triangle, is the following.



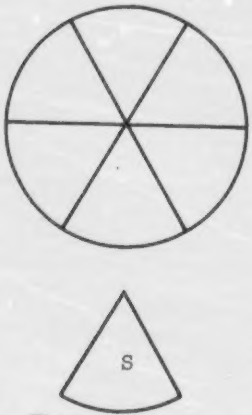
Another pattern is the following, using the isosceles right triangle.



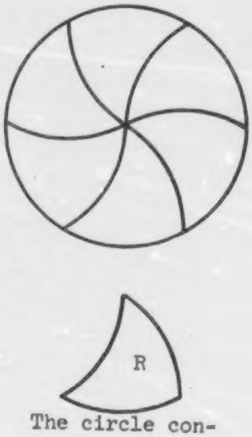
If one picks out one of the triangular regions to examine, he finds there is a square on the hypotenuse (longest side) made of four triangular tiles, while on each of the shorter sides is a square formed from two tiles. This shows a special case of the famous theorem (of Pythagoras) that for any right triangle the area of the square on the hypotenuse is the sum of the areas of the squares on the shorter sides.

3. Some regions whose boundaries are not all straight lines may be covered by non-overlapping units whose boundaries are not necessarily all straight lines nor curved lines.

These figures are examples.



The circle contains six units



The circle contains six units

VI-6. Doubling Edges of Rectangles

Objective: To observe the effect on perimeter and area of a rectangle if the sides are doubled.

Vocabulary: (No new words.)

Materials: Nine or more congruent rectangular sheets of paper with provision to mount them on the chalkboard or other convenient place.

Suggested Procedure:

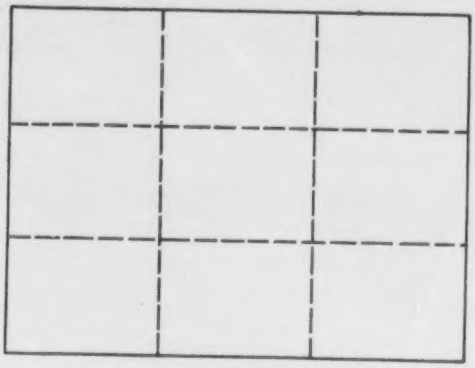
Invent some story that will introduce rectangular regions, one of which has sides twice as long as the other. For example, you might tell about a man who had two rectangular flower beds, a small one in his front yard and a larger one in his back yard. The sides of the flower bed in the back yard are just twice as long as those in the front yard. He put up a fence around his front flower bed and found it took 20 feet of fence. Then ask how many feet of fence will be needed for the flower bed in the back yard. Be sure the children understand that they are comparing the perimeters of the two rectangles, but do not try to settle any differences of opinion. Simply make note of the suggestions. (Quite possibly all of the children will agree that the correct answer should be 40 feet.)

Continue with the story of the flower beds. Indicate that each year the man has to spade up the flower beds to get them ready to plant. He finds that it takes him an hour to spade up the front bed. Ask how much time it will take for the back bed. Again do not try to settle any differences of opinion, but make a note of the different suggestions. Bring out by questioning that he must dig up the whole region so that we are really just asking to compare the areas of the two regions.

Indicate that we will make use of coordinate systems to look at the question. Ask the children if they remember a way we found to draw a figure whose sides were twice as long as those of another figure. (Multiply the coordinates by 2.) Now have them turn to page 344 in the pupil's book. The first part of this problem is intended to lead the pupil to the conclusion that doubling the sides of a rectangle doubles the perimeter (as one might guess). The second part of the problem is intended to have the pupil see that the area is not doubled, but is instead multiplied by 4. It would be well to check this by having the pupil actually find the areas of the two regions, i.e., 40 square units and 160 square units, but it seems that the real insight comes in seeing geometrically that it takes 4 copies of region ABCD to cover region PQRS.

These insights can now be related to the story of the flower gardens where the children can conclude that it would take 40 feet of fence for the back garden but would take 4 hours to spade it up.

Another experience to reinforce and extend these insights is the following. Have one of the rectangular regions mounted on the board. Now have the children place congruent regions to construct one with sides twice as long. It is clearly evident that it takes four sheets of paper in all, so again we see that doubling the sides multiplies the area by 4. Now suggest trying to place further congruent copies to construct a region whose sides are three times as long as the original one. The resulting figure will look something like this.



By examining this figure, notice that the perimeter is exactly three times that of the original region but that the area is 9 times as great. If interest and space permit, this could be extended to multiplying lengths by 4 and so on. Some children may even reach the (correct) conjecture that if the lengths of the sides are multiplied by K, then the perimeter is multiplied by K and the area is multiplied by K^2, though they will find it hard to verbalize.

Some child may notice after he completes the drawing on page 345 in the pupil's book that rectangles ABCD and PQRS have an intersection, in set terminology, which has an area of 4 square units. If so, tell him that this fact will be explored somewhat further in Section 8.

Doubling Sides of Rectangles

1. Draw rectangle ABCD if the numbers describing the points are
   A(1, 2) B(4, 2) C(4, 6) D(1, 6).

   Double these numbers to get points
   P(2 , 4) Q(8 , 4) R(8 , 12) S(2 , 12).

   Draw rectangle PQRS.

   The perimeter of ABCD is 14 units.
   The perimeter of PQRS is 28 units.

   The perimeter of PQRS is 2 times as large as the perimeter of ABCD.

   Write the equation used to find the area of region ABCD.

   $4 \times 3 = 12$

   Write the equation used to find the area of region PQRS.

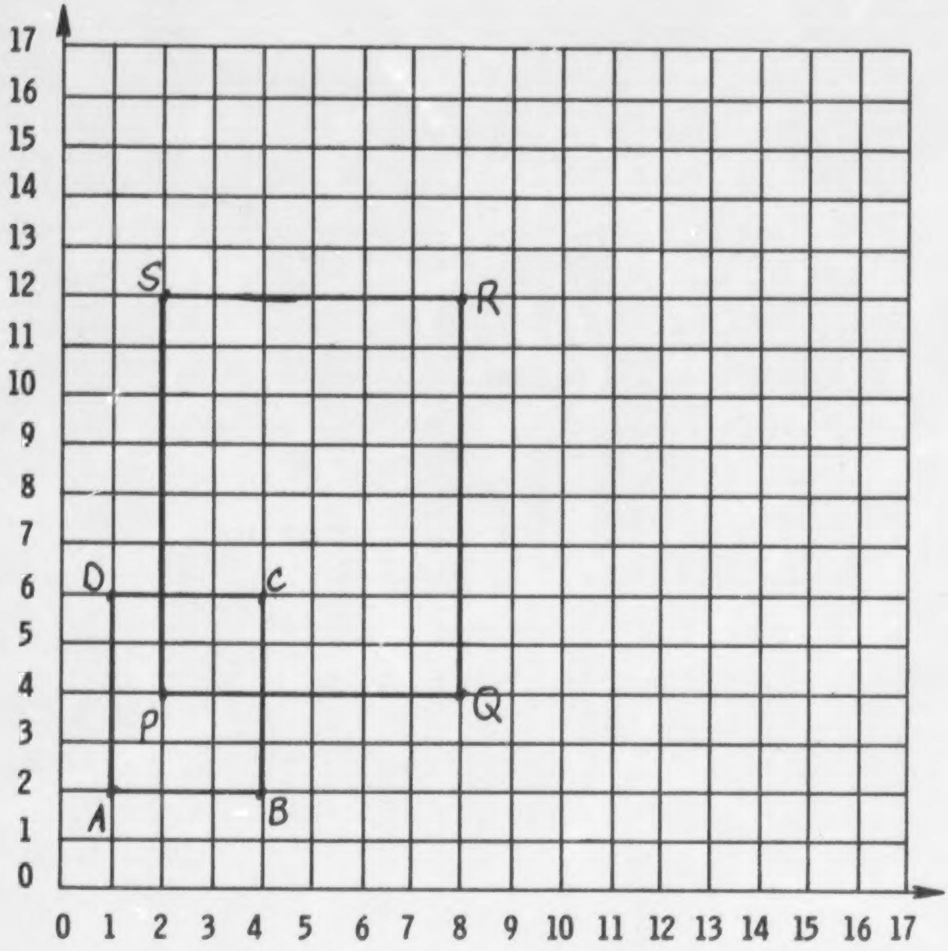
   $8 \times 6 = 48$

   The area of region ABCD is 12 square units.
   The area of region PQRS is 48 square units.

   The area of PQRS is 4 times as large as the area of ABCD.

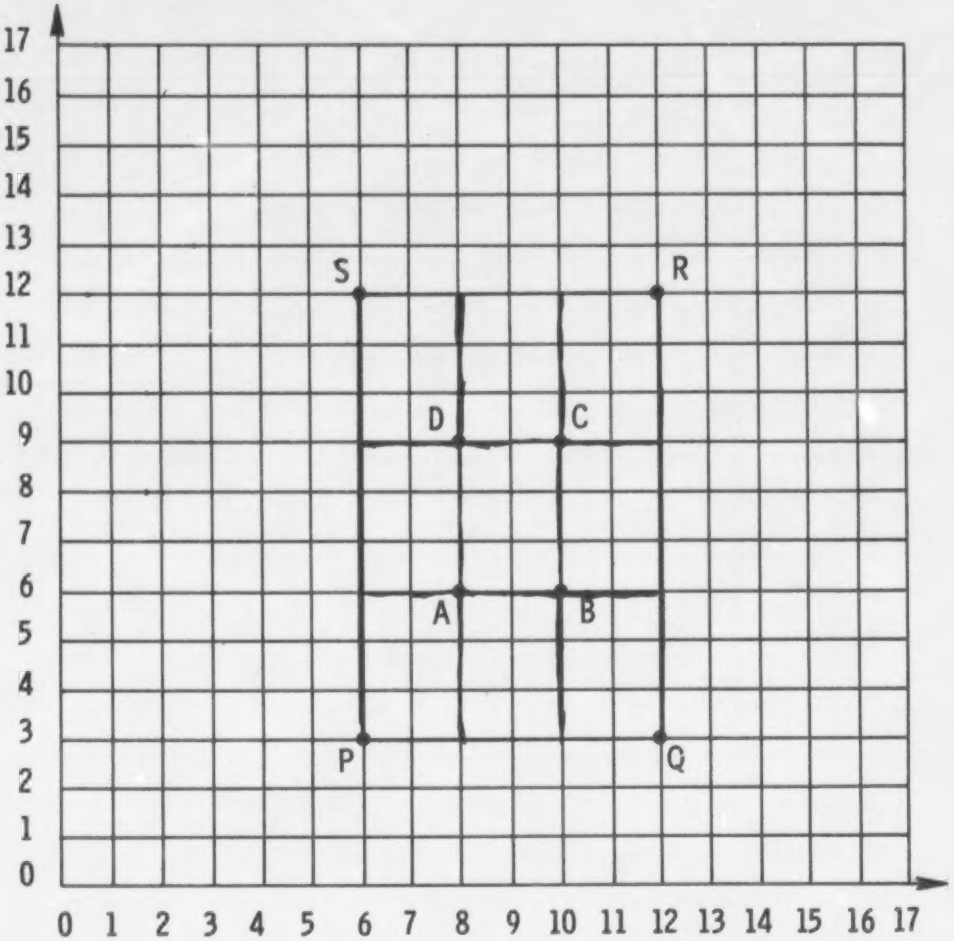
Doubling Sides of Rectangles

1.


S
R
D
C
P
Q
A
B
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Tripling Sides of Rectangles

2.


S
R
D
C
A
B
P
Q
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

The numbers describing A, B, C, D, are

A(8 , 6), B(10 , 6), C(10 , 9), D(8 , 9).

The numbers describing P, Q, R, S, are

P(6 , 3), Q(12 , 3), R(12, 12), S(6 , 12).

Draw rectangle PQRS.

Tripling Sides of Rectangles

Look at the facing page.

2. ABCD and PQRS are similar rectangles.

   Write the equation used to find the perimeter of ABCD.

   $2+3+2+3=10$

   The perimeter of ABCD is 10 units.

   Write the equation used to find the perimeter of PQRS.

   $6+9+6+9=30$

   The perimeter of PQRS is 30 units.

   The perimeter of PQRS is 3 times the perimeter of ABCD.

   Extend $\overline{AB}$ and $\overline{CD}$ to meet $\overline{SP}$ and $\overline{QR}$.

   Extend $\overline{AD}$ and $\overline{BC}$ to meet $\overline{RS}$ and $\overline{PQ}$.

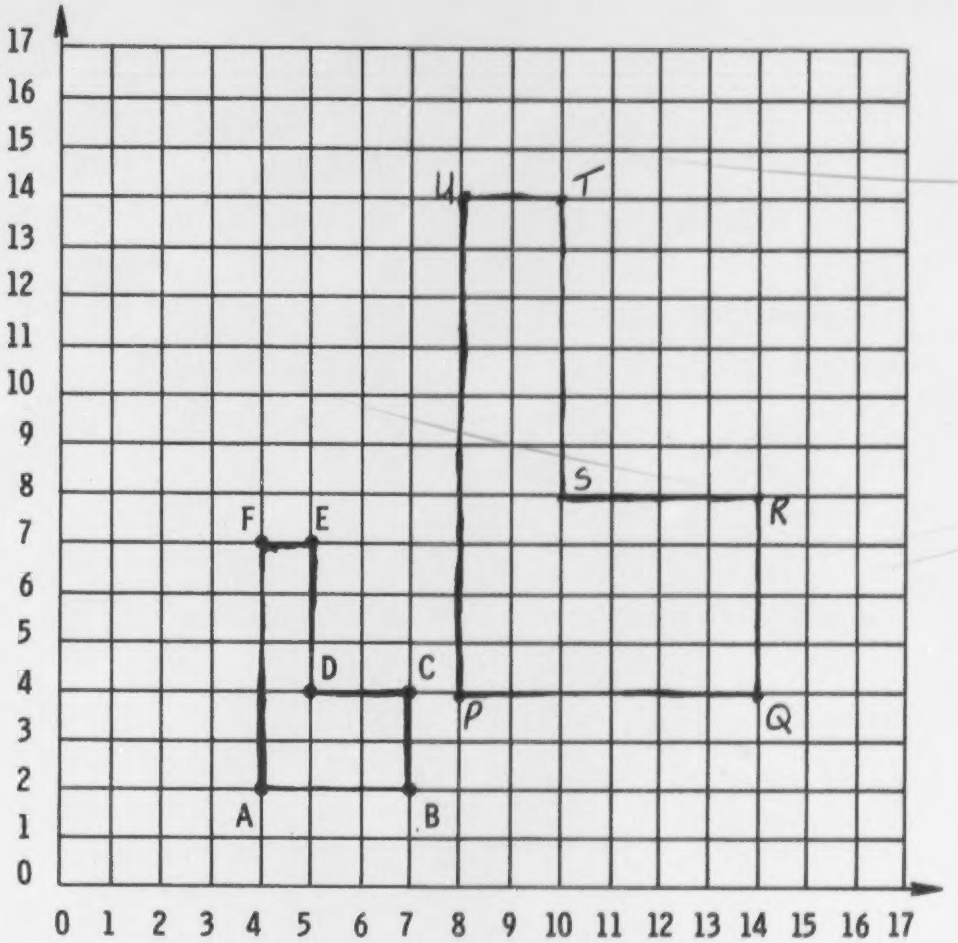
   These lines divide region PQRS into 9 smaller regions.

   Each region is congruent to ABCD.

   The area of region PQRS is 9 times the area of region ABCD.

Doubling Sides of Regions

3.


17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
A
B
C
D
E
F
P
Q
R
S
T
U

Doubling Sides of Rectangles

3. ABCDEF is a simple closed curve.

It is the edge of a region called **R**.

The perimeter of ABCDEF is 16 units.

The area of **R** is 9 square units.

Double the numbers describing points A, B, C, D, E, F.

Label the new points P, Q, R, S, T, U.

PQRSTU is the edge of a region called **S**.

The perimeter of PQRSTU is 32 units.

The area of **S** is 36 square units.

The perimeter of PQRSTU is 2 times as long as the perimeter of ABCDEF.

The area of **S** is 4 times as large as the area of **R**.

Doubling Sides of Rectangles

4. Two sides of a rectangle are 5 inches and 7 inches.

   The area is 35 sq. in. .

5. The area of a rectangle is 48 square inches. One of the sides is 6 inches long. Write an equation for the number n of inches in the other side.

   6 x n = 48 .

   The length of the other side is 8 in. .

6. The area of a rectangle is 36 square inches. One of the sides is 6 inches. Write an equation for the number n of inches in the other side.

   6 x n = 36

   The length of the other side is 6 in. .

   This rectangle is a square .

7. The area of a square is 25 square feet. Write an equation for the number n of feet in each side.

   n x n = 25

   The length of each side is 5 ft. .

VI-7. Congruence and Area

Objective: To observe that congruent figures have equal areas.

To see that area is additive.

To apply these ideas to determine certain areas.

Vocabulary: Overlapping, non-overlapping.

Materials: Colored chalk.

Suggested Procedure:

Draw (or mount) on the board two congruent rectangular regions. Be sure the children understand the regions are intended to be congruent. Ask whether the two regions have the same area. There will probably be unanimous agreement, but have the pupils explain *why* this should be true. (The essential point to be developed is that if two regions are congruent, a copy of one will just fit on the other. But this means that if one of them is covered by a set of unit square regions, the same set of unit regions arranged in the same way must just fit on the other one. Hence, the areas must be the same.) Write this conclusion on the board.

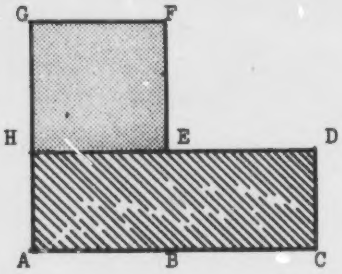
Congruent regions have equal areas.

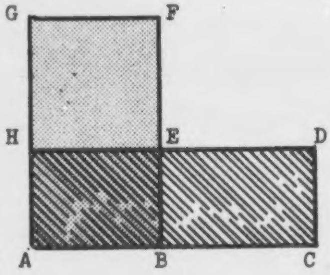
Pupil's book, pages 352-353:

Work with the class as the notation becomes involved. Then ask the children to explain the results: that is, in both cases the larger region is a union of two smaller ones. Yet in one case the number of square units in the larger could be found by adding the numbers for the two smaller regions, and in the other it could not.

Elicit such statements as that, in the second case, "the regions overlap" or "the regions have points in common". Examine this carefully with the class. Draw two copies on the board and have the small regions colored in contrasting colors of chalk. The figures would be somewhat like the ones below.

Anything which is doubly colored would belong to both small regions, i.e., would be in their intersection. In the first case only $\overline{HE}$ is doubly colored, so the intersection is just this segment. The segment is part of the edges of the regions.





Note, however, that the intersection of the regions is not empty. In the second case, on the other hand, the whole rectangular region ABEH is doubly colored. That is, the intersection contains interior points of the regions. This is the trouble. When we add the number of squares for ACDH and for ABFG, any squares in ABEH are counted twice. Two regions like this which have common interior points are called overlapping. Show overlapping regions with sheets of paper, one partly over the other.

From this discussion lead the pupils to the understanding that if a region is divided into sub-regions which do not overlap, the number of square units in the region is the sum of the numbers of square units in the two parts. We call such regions non-overlapping regions.

Now call to the pupils' attention (if they have not already noticed it) that we have talked about area only for rectangular regions or ones that are easily broken into rectangular regions. For example, we have done nothing with a triangular region such as that on page 354 in the pupil's book. Have the pupil turn to this page and let him notice that, to have fitting along $\overline{BC}$, there would have to be a lot of cutting of the unit square regions, and it is not clear how many whole square units this would use up. Explain that questions like this will be discussed in a later grade, but that this problem will show one way of working with them. Now go on with the questions on page 356 in the pupil's book. Where the question "Why?" is asked, have some child explain orally.

Supplementary pages; 358-370:

Further work is given on testing for congruent figures. The idea of congruent regions is used to compare areas.

Congruence and Area

1. Look at the facing page.

See the simple closed curve ACDEFG.

This is the edge of a region.

The area of this region is 33 square units.

Find the point whose coordinates are (7, 8). Mark it H. Draw $\overline{HE}$.

The area of region ACDH is 21 square units.

The area of region HEFG is 12 square units.

Is region ACDEFG the union of region ACDH and region HEFG?

Yes.

Find the point whose coordinates are (9, 5). Mark it B. Draw $\overline{BE}$.

The area of region ACDH is 21 square units.

The area of region ABFG is 18 square units.

Is region ACDEFG the union of region ACDH and region ABFG?

Yes.

Is the number of square units in ACDEFG the sum of the numbers ACDH and ABFG? No.

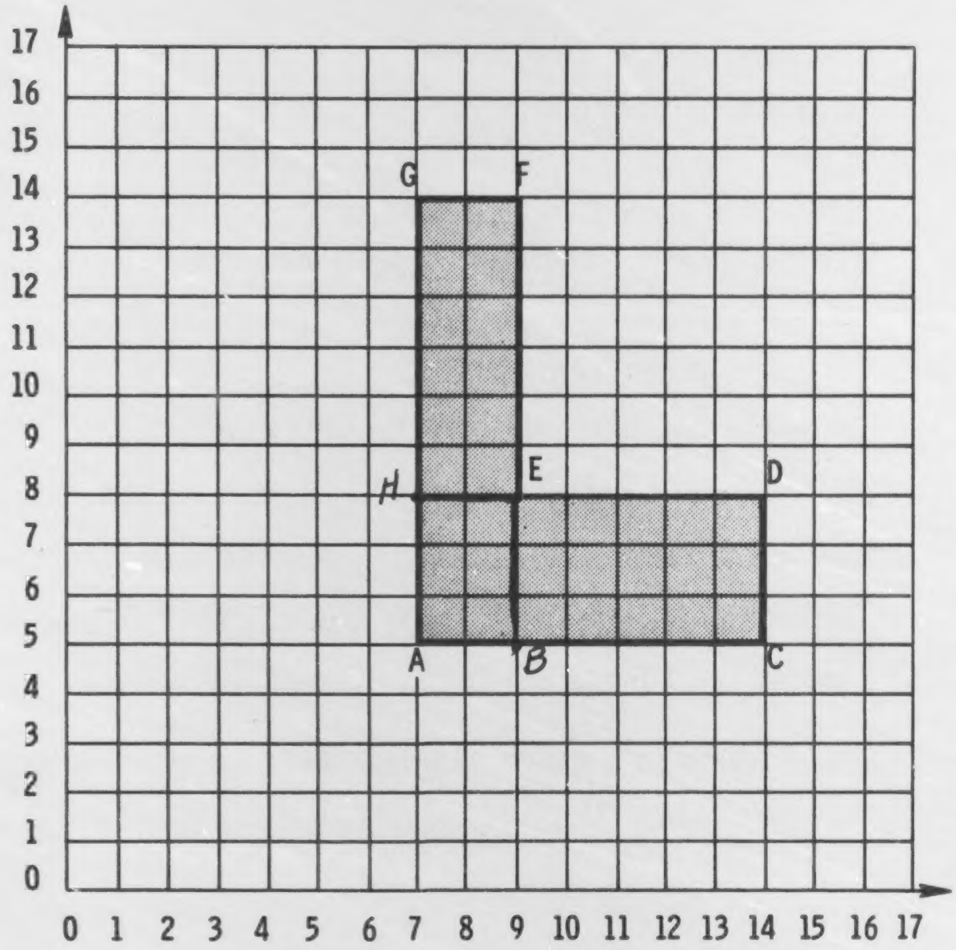
What is the area of the region ABEH? 6 square units.

Subtract this number from the sum of the numbers for the areas of ACDH and ABFH. The difference is 33.

How does this number compare with the number for the area of the region ACDEFG? It is the same.

Congruence and Area

1.


17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
G
F
H
E
D
A
B
C

Congruence and Area

2. Look at the facing page.

Find point D(13, 9).

Draw $\overline{CD}$ and $\overline{BD}$.

ABDC is a rectangle.

The area of region ABDC is 60 square units.

Is $\triangle$ABC congruent to $\triangle$DEC? Yes.

Do regions ABC and DEC have the same area? Yes. Why?
Answers will vary.

Does the number of square units in region ABDC equal the sum of the numbers for ABC and DEC? Yes. Why? Answers will vary.

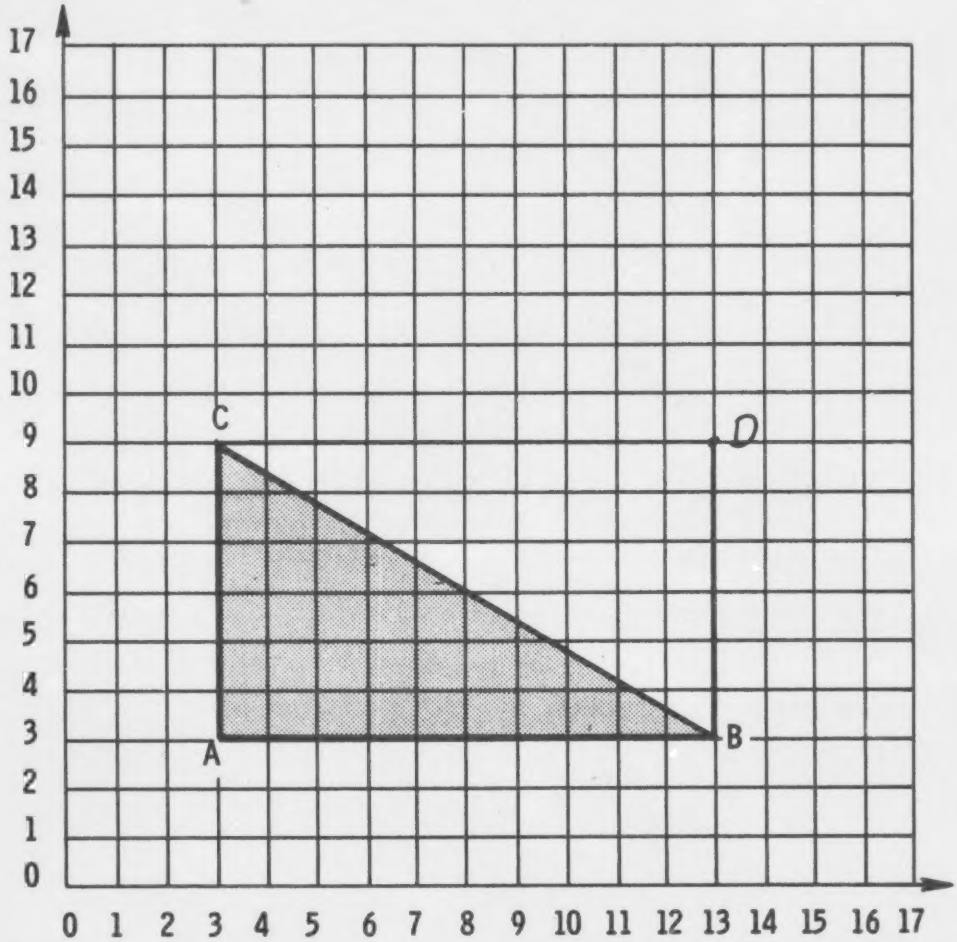
The number of square units in region ABDC is twice the number for region ABC. Why? Answers will vary.

The area of region ABC is 30 square units.

This number is one half of that for the area of the rectangle ABDC.

Congruence and Area

2.


17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
C
D
A
B

3. Look at the facing page.

ACDF is a quadrilateral.
It is the edge of a region.
We shall find the area of region ACDF.

The area of region ABEF is the same as the area of region CPQD.
Why? ABEF is congruent to CPQD.

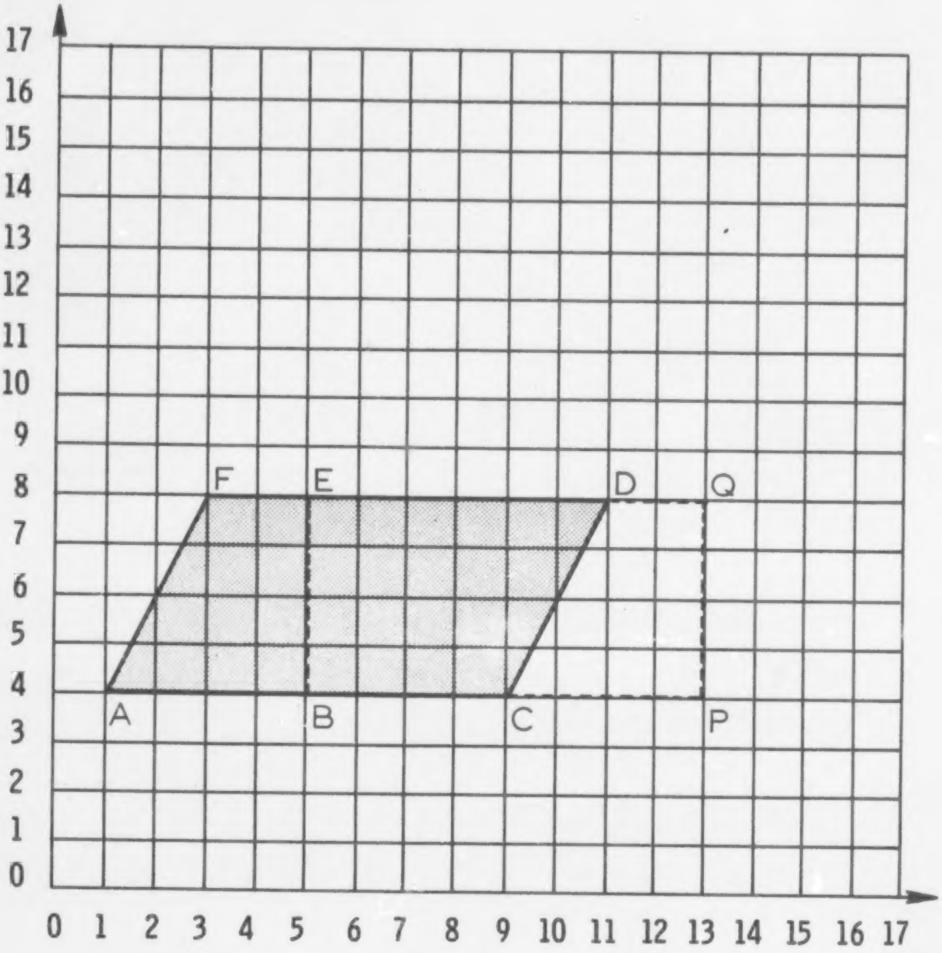
The area of region ACDF is the same as the area of region BPQE.
Why? Answers will vary.

The area of region BPQE is 32 square units.
Why? 4 x 8 = 32

The area of region ACDF is 32 square units.

Congruence and Area

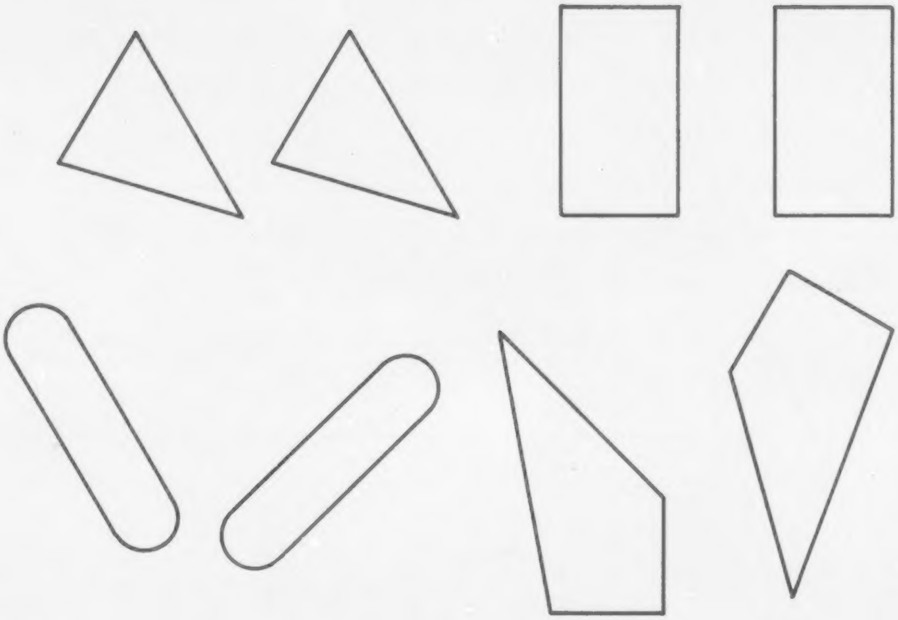
3.


17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
F
E
D
Q
A
B
C
P

SUPPLEMENTARY

Further Work with Areas

You know what is meant by congruent figures. Below are some pairs of congruent simple closed curves.



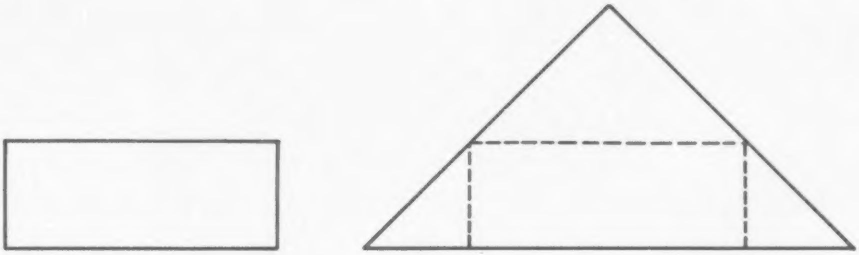
A simple closed curve is the boundary of a region.

If two simple closed curves are congruent, are their regions congruent? Yes.

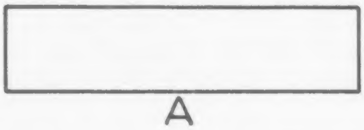
Does the picture show pairs of congruent regions? Yes.

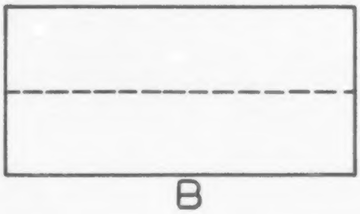
2. We know that congruent regions have equal areas.

   If a tracing of one region can be made to fit entirely inside another region, then the first figure has a smaller area than the second. For example, the rectangular region below has a smaller area than the triangular one because a tracing of the rectangle can be made to fit inside the triangle as shown by the dashed line.



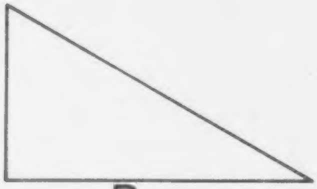
3. Exactly two regions congruent to A will fit into region B .



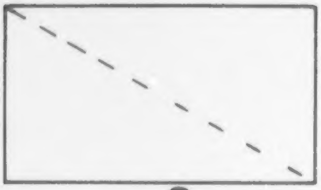


   Then region B has exactly twice the area of region A .

4.



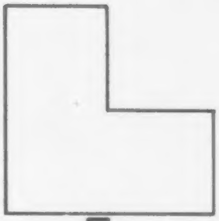
R



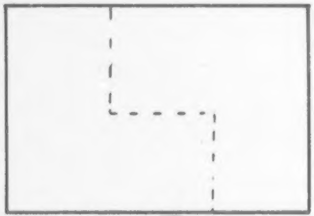
S

Show by tracing and fitting that region S has exactly twice the area of region R .

5. Show by tracing and fitting that region E below has exactly twice the area of region F . Draw a dashed line to show how you did the fitting.

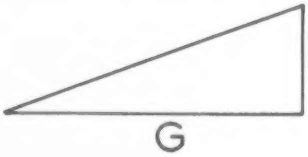


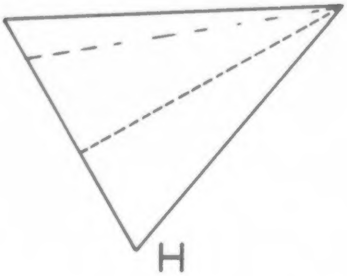
F



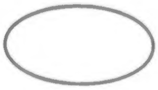
E

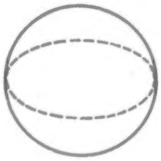
6. Look at regions G and H below. It is possible to fit two regions congruent to G inside H so that there is space left over. This shows that triangle H has more than twice the area of triangle G. Show by dashed lines how to do the fitting.

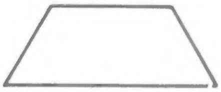


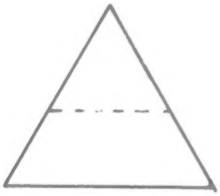


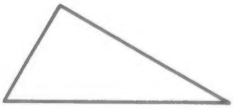
7. Use tracing paper to show how the left-hand region in each pair below can be made to fit inside the right-hand figure.

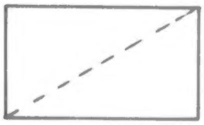






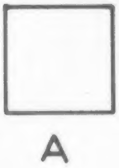


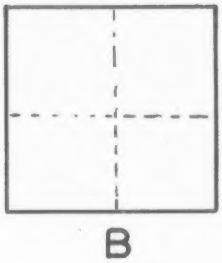




In each example, fit as many regions congruent to region A inside B as you can. Complete the sentences that tell about the areas.

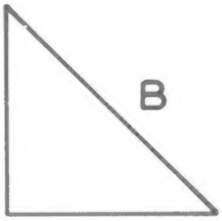
1.

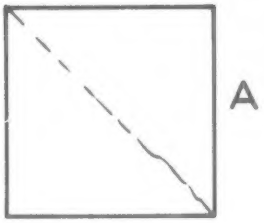




The area of B is exactly 4 times the area of A .

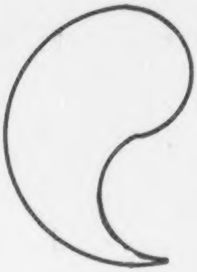
2.

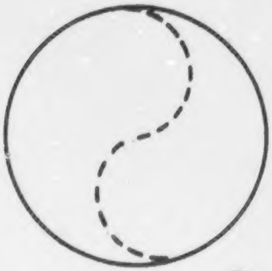




The area of A is exactly 2 times the area of B

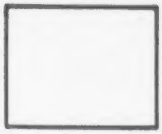
3.

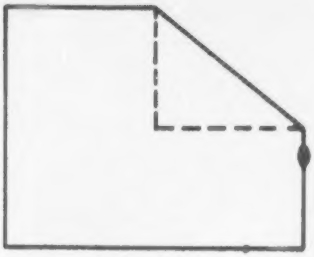




The area of B is exactly 2 times the area of A .

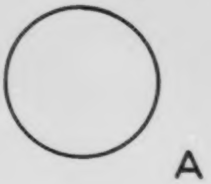
4.

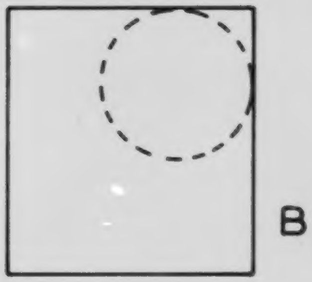




The area of B is more than 3 times the area of A .

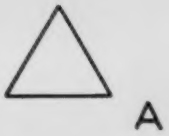
5.

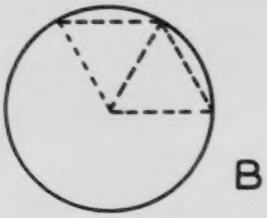




The area of B is more than 2 times the area of A .

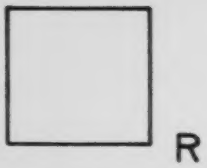
6.

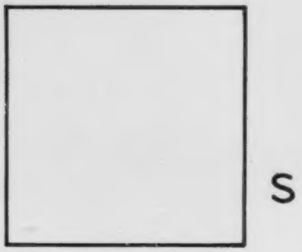




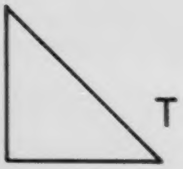
The area of B is more than 6 times the area of A .

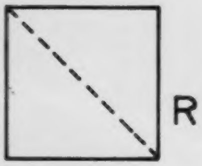
7. Look at these two square regions.

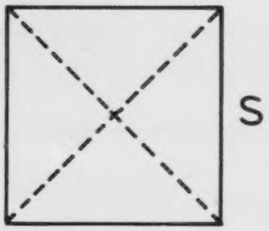




We can fit one square region congruent to R inside S with some space left over, but we cannot fit two regions congruent to R inside S without overlapping. The fact is, however, that the area of S is exactly 2 times the area of square R . Can you think how we could know this? Here is a simple way to show it.

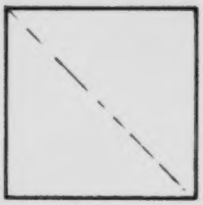


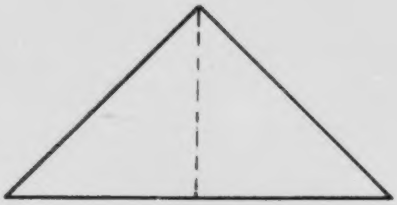




The area of R is exactly 2 times the area of T . The area of S is exactly 4 times the area of T . Four things are twice as many as two things. Therefore, the area of S is twice the area of square R .

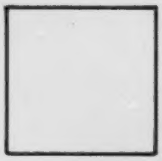
How can you show that this square and this triangle have the same area?



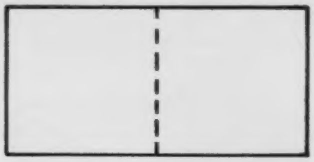


Answer: Answers will vary.

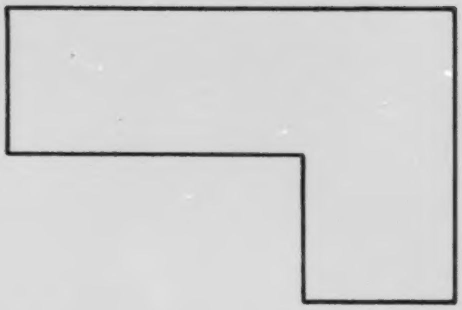
Here is a square region whose sides are one inch long.



We say that the area of this region is one square inch.



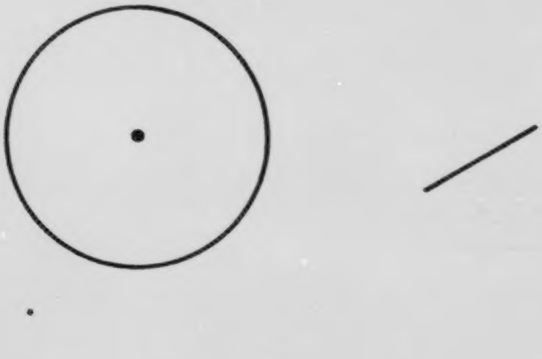
The area of this rectangular region is two square inches.



The area of this region is 4 square inches.

To finish this section let's try to find out something about the area of circular regions. Below is a circle. The point in the middle is called the center of the circle. Next to the circle is a line segment. Make a tracing of the line segment and put one end of your tracing on the center of the circle. Where does the other end of the segment lie? On the circle.

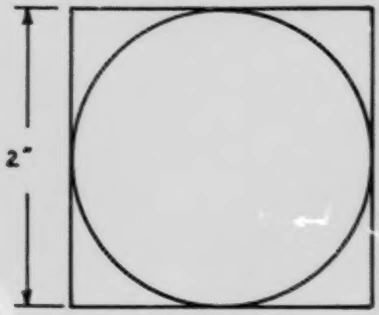
The length of the segment is called the radius of the circle.



The radius of this circle is exactly one inch.

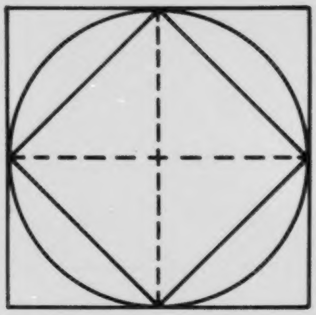
What can we find out about the area of the region enclosed by the circle?

To begin with, we can put our circle inside a square whose sides are two inches long.



The area of the square region is ___4___ square inches.
The area of the circular region is less than ___4___ square inches.

Now we can put another square inside the circle.



Can you find the area of the inside square region? 2 sq. in.

Hint: Divide the region into four triangular regions.

Does this remind you of something you did before? Yes

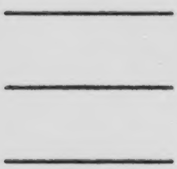
The area of the circular region is more than 2 square inches.

We have seen that the area of the circular region whose radius is one inch is more than 2 square inches, but less than 4 square inches.

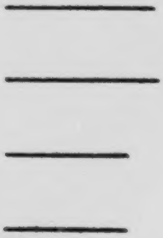
Later on you will learn that the area of this region is just a little more than 3 square inches.

*Line Segments

1. Here are three line segments. Draw a triangle with sides congruent to these segments.



2. Here are four line segments. Draw a quadrilateral with sides congruent to these segments, and draw it so that it will fit inside the triangle you just drew.



3. I have two triangles A and B. Every side of A is longer than every side of B, and yet I cannot fit triangle B inside triangle A! How is this possible? Can you show by a drawing?

VI-8. Gauss sums and Euler routes

Objective: To introduce more able pupils (or more able classes) to some simple combinatorial geometry.

Vocabulary: (No new words.)

Background: Once when Carl Friedrich Gauss was a small boy, he and his classmates were required, as a punishment, to add up all the numbers from 1 to 100. Little Gauss quickly saw that he could save himself a lot of trouble if he added the numbers in these pairs: 0 + 100, 1 + 99, 2 + 98, etc. On pupil pages 371-373 we present a simple geometric problem involving sums of the form

$$0 + 1 + 2 + \ldots + n.$$

At the end we give a hint which, we hope, may lead some pupils to rediscover Gauss's neat trick.

Leonhard Euler once proposed to walk over each of Koenigsberg's seven bridges just once and return to his starting point. He soon found this to be impossible. On pupil pages 374-377 the present a sequence of problems designed to lead the pupil to a simple insight about problems of the Koenigsberg bridge type.

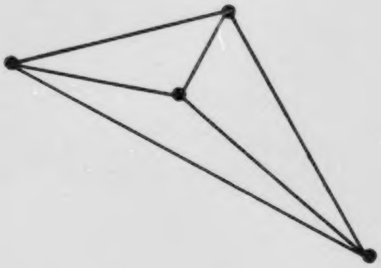
Both of these problem sets involve a kind of geometry which, in contrast to the congruence geometry of Chapter I, is not concerned with the exact shape of things. This kind of geometry is called Topology.

Suggested Procedure:

The pupil pages 370-377 are intended primarily for independent use by individual children. Page 370 gives a little practice in thinking intuitively about geometrical objects. Pupil pages 371 - 373 are intended to give the pupil an inkling of argument by induction. The important thing for him to see is how to go from the case of one point to the case of two points, from the case of two points to the case of three points, and so forth, and finally how to go from the case of n points to the case of $n + 1$ points where n is any number. This technique, induction, is one of the commonest and most powerful techniques in mathematics. Pupil pages 374-377 concerning Euler routes should lead the pupil to realize that a map cannot have an Euler route if any of its towns has an odd number of roads leading from it. No doubt many children who see this will jump to the conclusion that an even number of roads leading from every town guarantees the existence of an Euler route. This is true, but it is not so easy to prove.

Line Segments and Sums

1. Here are four points:

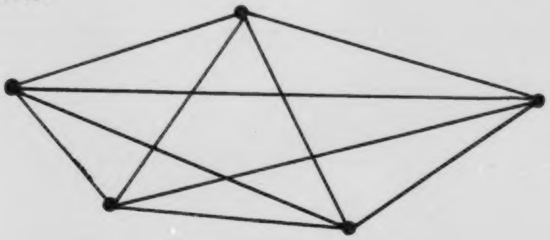


Join every pair of points with a line segment.

How many segments did you draw? 6

Find the sum: 3 + 2 + 1 + 0 = 6

2. Here are five points:

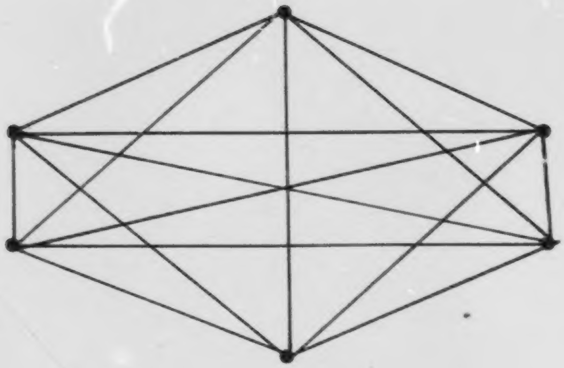


Join every pair of points by a line segment.

How many segments did you draw? 10

Find the sum: 4 + 3 + 2 + 1 + 0 = 10

3. Here are six points:



Again, join every pair of points by a line segment.

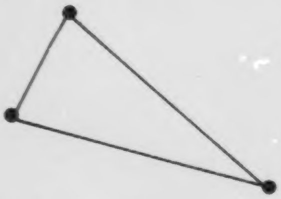
How many segments this time? 15

Find the sum: $5 + 4 + 3 + 2 + 1 + 0 =$ 15

Was the sum the same as the number of segments every time? __________

If not, go back and check your work.

4. Now try it for three points. Figure out what numbers you should add.



Number of segments: 3

2 + 1 + 0 = 3

Does it work? Yes. Does it work for two points? Yes.

For one point? Yes. Now add a fourth point to the three you have up above. How many new segments can you draw? 3

What new number should you add to the sum? 3

Now think about what you have done and try to explain why the sum should always be the same as the number of segments.

Answers will vary.

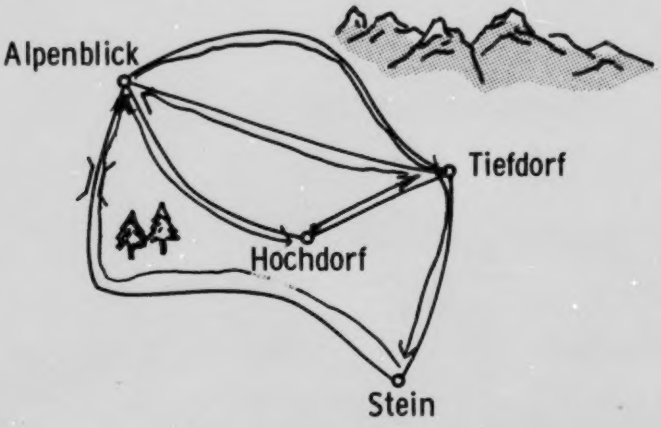
5. Without drawing anything figure out how many line segments it would take to join all pairs in a set of ten points:

9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 45

If you rearrange these numbers in the right way the addition is very easy.

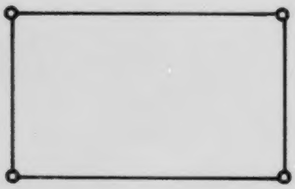
Routes

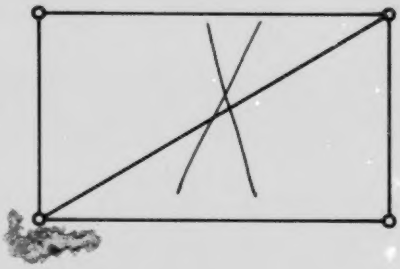
I am staying in Hochdorf, a village high in the Alps. I would like to take a trip starting and ending at Hochdorf and passing over every road exactly once. Trace out a route for me.

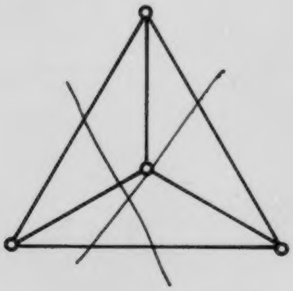


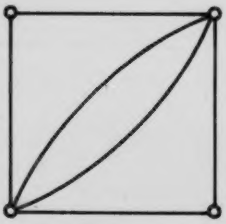
1. We call a route which starts and ends at the same point and passes over every road exactly once an Euler route after the famous Swiss mathematician Leonhard Euler (pronounced "Oiler") who first worked problems of this kind. Find an Euler route starting and ending at Stein. If there is an Euler route starting and ending at one point is there necessarily an Euler route starting and ending at every other point? Yes. Why? Answers will vary.

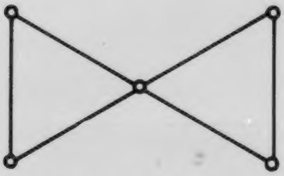
2. Try to find an Euler route for each of the maps below. Start at any point you like, but be sure to end at the same point. Put a big X on each map which has no Euler route.

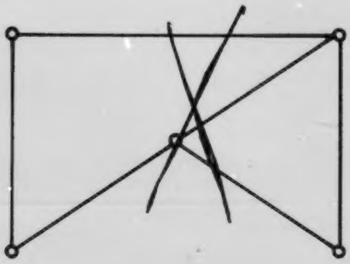






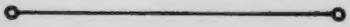




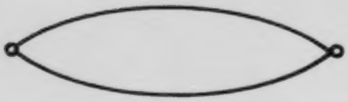


Can you think of any simple rule to tell you when a map will have an Euler route and when it will not? Look back at the maps above.

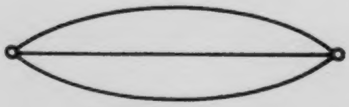
3. Now look at these maps. Mark with X those maps which have no Euler route.



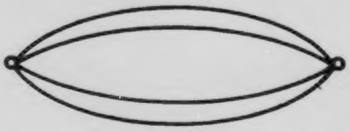
X



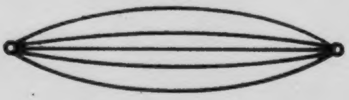
____



X



____

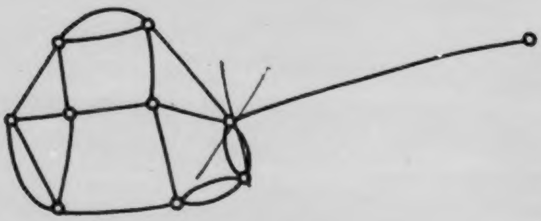


X

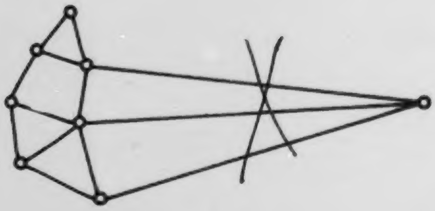
What do you notice? Answers will vary.

Anything about even and odd... ?

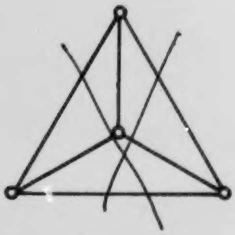
If you look carefully you can tell right away that each of these maps has no Euler route. Explain how on the line under each map.



Answers will vary.



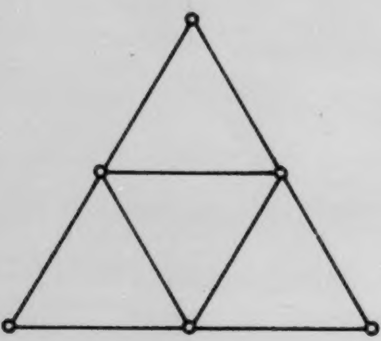
Answers will vary.

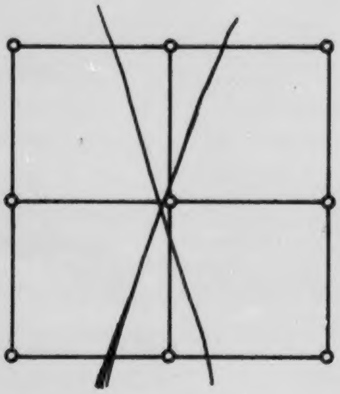


Answers will vary.

4. Now can you think of a simple rule to tell you when a map has no Euler route? It has no Euler route if any of its towns has an odd number of roads leading from it.

Try your rule on these:





Chapter VII

MULTIPLICATION, QUOTIENTS, AND DIVISION

Background

Associativity of multiplication. We have defined multiplication in terms of arrays: 3×8, for instance, is the number of elements in an array of 3 rows of 8 elements each. Once we are familiar with this definition, it is not necessary to think of actually arranging the set of elements of this array in rows. We may think of 3×8 as being simply the number of members in the union of 3 disjoint sets of 8 members each.

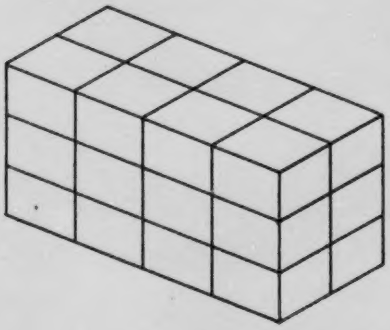
In terms of this idea it is easy to justify the associative property of multiplication, according to which, for example,

$$(2 \times 3) \times 4 = 2 \times (3 \times 4).$$

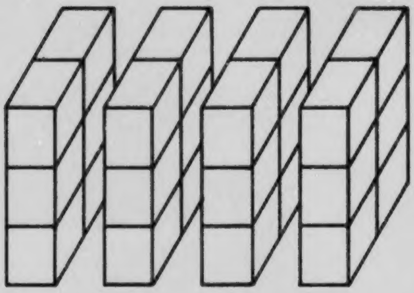
(Recall that we have already noted much earlier the associative property

$$(2 + 3) + 4 = 2 + (3 + 4)$$

of addition.) We begin by imagining a stack of blocks which is 2 blocks wide, 3 blocks high, and 4 blocks long.



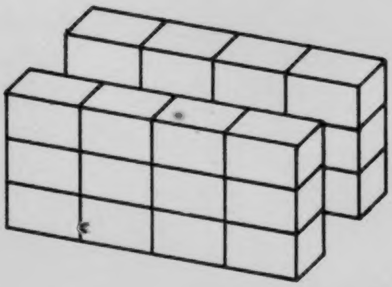
We may think of this set of blocks as the union of 6 blocks in each of 4 sets.



This shows that the number of blocks in the whole stack is 6×4, which, since $6 = 2 \times 3$, is the same as

$$(2 \times 3) \times 4.$$

On the other hand, we may also think of this same set of blocks as being the union of 2 sets of 3×4, or 12, blocks each:



This shows that the number of blocks in the whole stack is

$$2 \times (3 \times 4).$$

We have now seen that the number of blocks in the whole stack is given both by $(2 \times 3) \times 4$ and also by $2 \times (3 \times 4)$. Hence

$$(2 \times 3) \times 4 = 2 \times (3 \times 4).$$

Distributivity of multiplication over addition. Multiplication may be distributed over addition, both "from the left", as in the example

$$4 \times (2 + 5) = 4 \times 2 + 4 \times 5,.$$

and "from the right", as in the example

$$(2 + 5) \times 4 = 2 \times 4 + 5 \times 4.$$

To see the first of these relations, we take a 4 by $(2 + 5)$ array, ie., a 4 by 7 array,

$$\begin{array}{ccccccc} x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \end{array}$$

and think of it as formed from a 4 by 2 array and a 4 by 5 array, as suggested below:

$$\begin{array}{cc|ccccc} x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \end{array}$$

This makes it clear that the number of elements in a 4 by $(2 + 5)$ array is the number of elements in a 4 by 2 array plus the number of elements in a 4 by 5 array. That is,

$$4 \times (2 + 5) =(4 \times 2)+(4 \times 5).$$

To see the relation

$$(2 + 5) \times 4 =(2 \times 4)+(5 \times 4),$$

we can similarly take a $(2 + 5)$ by 4 array and think of it as formed from a 2 by 4 array and a 5 by 4 array, thus:

x x x x
x x x x

x x x x
x x x x
x x x x
x x x x
x x x x

Or, alternatively, we may use commutativity to replace $4 \times (2 + 5)$ by $(2 + 5) \times 4$, 4×2 by 2×4, and 4×5 by 5×4, in the equation

$$4 \times (2 + 5) =(4 \times 2)+(4 \times 5.)$$

The result of making these three replacements is the desired equation

$$(2 + 5) \times 4 =(2 \times 4)+(5 \times 4.)$$

Quotients. There is a strong parallel between the relationship of sums and differences on the one hand and the relationship of products and quotients on the other. To make this completely clear, we begin by considering a typical addition equation, say

$$5 + 4 = 9.$$

We may think of this as saying that 5 is the number such that this number plus 4 gives the sum 9; and we may express this fact by writing 5 as the difference of 9 and 4:

$$5 = 9 - 4.$$

Now consider any multiplication equation, say

$$5 \times 4 = 20.$$

We may think of this as saying that 5 is the number such that this number times 4 gives the product 20. We shall express this fact by writing 5 as the quotient of 20 and 4, thus: $5 = 20 \div 4$.

Remember that for the present we are dealing only with the whole numbers, that is, the numbers 0, 1, 2, 3, 4, etc. As long as this is true there will be some addition questions, like

$$n + 4 = 3,$$

which have no answers. That is, there is no whole number such that a number plus 4 is 3. (Later on, of course, the negative numbers will be introduced, and then -1 will be the answer to the question $n + 4 = 3$. That is, -1 will then be the difference $3 - 4$.)

There is a similar situation as regards multiplication. As long as we are dealing only with the whole numbers, there will be some multiplication question, like

$$n \times 4 = 21$$

which have no answers. That is, there is no whole number such that this number times 4 is 21. Later on, in the chapter on rational numbers, $\frac{21}{4}$ will be identified as the answer to the question $? \times 4 = 21$. That is, $\frac{21}{4}$ will then be the quotient $21 \div 4$.)

Division. We have just seen that with respect to certain "questions without answers", the situation for multiplication closely parallels the situation for addition. But there is an important distinction.

It is easy to see in advance just when it is that an addition question, like

$$n + 4 = 3,$$

is going to have no whole number answer; it is when the number "on the left" (4, in this case) is greater than the number "on the right" (3).

For multiplication, however, it is not so easy to see in advance whether or not a question like, say,

$$n \times 7 = 313$$

is going to have a (whole-number) answer. In fact, many third grade children may not even be able to see in advance whether or not the question

$$n \times 4 = 21$$

has a whole-number answer.

Questions such as this, for which there are no answers *within the set of whole numbers*, are deferred until Chapter 9 after consideration has been given to the rational numbers (Chapter 8). Here in Chapter 7 we restrict our work to questions such as

$$n \times 4 = 20$$

for which there are whole-number answers.

VII-1. Multiplying with 10

Objective: To extend the multiplication concept to products that are multiples of ten.

Vocabulary: (No new words.)

Materials: Manipulative materials, 3 by 10 array of objects.

Suggested Procedure:

Review thinking of multiples of ten in more than one way by asking how to rename 70 as tens. (7 tens.) Continue with many other multiples: 200, 360, 290, 140, 70, 250, 450, 100, etc.

Ask for another way to think about 3 tens. (30.) Ask if someone can make an array showing 3 tens. (3 rows of 10 elements each.) This may be drawn on the chalkboard or arranged with objects. Ask what multiplication equation could be made for the array. ($3 \times 10 = 30$.) State that 3×10 is another way of saying 3 tens. Ask what 4×10 is, and if the answer is 4 tens, have the number renamed as 40. Continue with 7×10, 2×10, 8×10, etc. Ask what 12×10 is. If there is hesitation, remind children that this is 12 tens and therefore 120. Give much practice with multiplying a number times 10, including numbers such as 45×10.

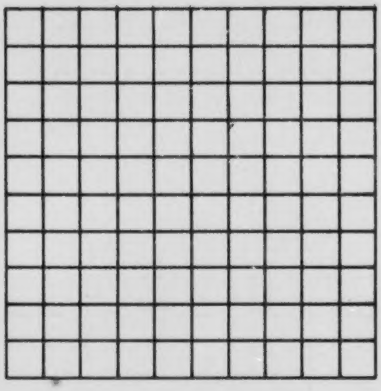
Continue practice, including examples like 24×10 and 10×9, 7×10 and 10×25. Occasionally refer to 8 and 10 as factors of 80.

Discourage any suggestion of "just adding a zero" to the number. If this arises, show that if a zero is added to 12 in the illustrative equation, for example, the sum of 12 and 0 is 12. If, however, children do not show all the steps in solving the equations on the pupil pages, allow them to continue to do the work mentally.

Use a similar procedure to show the use of multiples of 100: 300 = 3 hundreds = 3 × 100; 4 × 100 = 400; 24 × 100 = 2400, etc.

Pupil's book, pages 378 - 381: These pages provide practice. In this chapter, children are expected to get the general idea of multiplication and division without necessarily mastering all the basic facts. The drill on and mastery of basic multiplication and division facts is delegated to fourth grade.

Multiplying with Ten



Use the array above to help you fill the blanks.

Is there an easy way when 10 is one of the numbers?

2 × 10 = 20

4 × 10 = 40

10 × 1 = 10

10 × 3 = 30

5 × 10 = 50

10 × 5 = 50

8 × 10 = 80

9 × 10 = 90

10 × 7 = 70

4 × 10 = 40

10 × 5 = 50

3 × 10 = 30

2 × 10 = 20

10 × 7 = 70

10 × 1 = 10

8 × 10 = 80

10 × 0 = 0

6 × 10 = 60

A Multiplication Table

Can you make a multiplication chart to include 10? Try it.

×	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Multiplying with 10 and 100

Fill the blanks:

45 × 10 = 450
10 × 35 = 350
72 × 10 = 720
12 × 10 = 120
7 × 10 = 70
9 × 10 = 90
10 × 50 = 500
10 × 39 = 390
4 ×100 = 400
6 ×100 = 600
9 ×100 = 900
6 × 10 = 60
10 × 84 = 840
3 × 10 = 30
24 × 10 = 240
18 × 10 = 180
37 × 10 = 370

17 × 10 = 170
10 × 72 = 720
100 × 7 = 700
5 ×100 = 500
100 × 35 = 350
27 × 10 = 270
64 × 10 = 640
10 × 53 = 530
10 × 47 = 470
68 × 10 = 680
6 × 10 = 60
8 × 10 = 80
10 × 11 = 110
20 ×100 = 2000
17 ×100 = 1700
100 × 82 = 8200
21 × 10 = 210

Multiplying with Ten and a Hundred

Complete:

40 = 4 × 10

30 = 3 × 10

60 = 6 × 10

70 = 7 × 10

80 = 8 × 10

50 = 5 × 10

120 = 12 × 10

360 = 10 × 36

450 = 45 × 10

700 = 7 × 100

620 = 62 × 10

840 = 84 × 10

1500 = 15 × 100

720 = 72 × 10

2700 = 27 × 100

3900 = 39 × 100

4500 = 45 × 100

6000 = 60 × 100

VII-2. Using multiplication to solve problems

Objective: To learn to solve problems using multiplication facts, with particular emphasis on equations of the form $a \times b = n$ and $n \times b = c$.

Vocabulary: (No new words.)

Materials: Manipulative materials, if needed.

Suggested Procedure:

Review multiplication facts by writing on the chalkboard examples such as the following, and ask children to name the number that completes each.

$4 \times 9 = \_\_\_$ $\qquad$ $4 \times 10 = \_\_\_$
$3 \times 6 = \_\_\_$ $\qquad$ $7 \times 10 = \_\_\_$
$5 \times 5 = \_\_\_$ $\qquad$ $10 \times 6 = \_\_\_$
$7 \times 3 = \_\_\_$ $\qquad$ $3 \times 100 = \_\_\_$

Read, "Four times nine is equal to what number?"

Ask children what each example suggests. For instance, 4×9 can be related to objects with 9 in each set, etc. Then ask children to make up a story problem that can be related to this example, such as, 9 chairs can be placed at a table. We want to put that many chairs at 4 tables. How many chairs do we need? Have them suggest other problems that might be related to this example as well as to other examples that you have selected.

Then write on the chalkboard equations such as the following:

$n \times 4 = 20$ $\qquad$ $n \times 10 = 40$
$n \times 5 = 15$ $\qquad$ $n \times 10 = 70$

$n \times 6 = 24$ $n \times 100 = 300$

$n \times 7 = 21$ $n \times 7 = 700$

$n \times 5 = 500$

etc.

Read "What number times four is equal to twenty?"

Ask the children to use their multiplication facts to complete these equations. Then ask children to show what these equations mean to them. For example, for $n \times 4 = 20$, they know that they have 4 objects in each set, that they have 20 objects altogether, but they do not know how many sets they have. Then ask that someone give a story problem that suggests this equation. (e.g., Bill had 20 cookies in bags. He had 4 cookies in each bag. How many bags of cookies did he have?) Continue the discussion not only asking for other problems for $n \times 4 = 20$, but also asking for problems for equations that you have given.

Then write some problems on the chalkboard (or present on a chart or by some other visual medium) so that the children may see the problems. The following are suggested. Undoubtedly you will want to supplement these with similar problems.

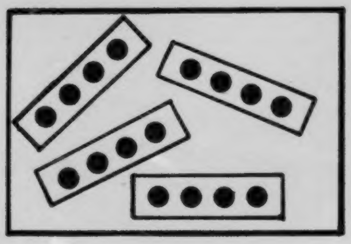
> Ann has 5 rows of stamps in her stamp book.
> There are 6 stamps in each row.
> How many stamps are in her stamp book?
> ($5 \times 6 = n$.)
>
> We can put 6 chairs in one row.
> We want to arrange chairs to have 4 rows.
> How many chairs will we need? ($4 \times 6 = n$.)
>
> We are going to take a trip.
> 5 children can ride in each car.
> There are 35 children in our class.
> How many cars will we need? ($n \times 5 = 35$.)

There are 8 candies in a package.
I need 32 candies for a party.
How many packages of candy will I need?
$(n \times 8 = 32.)$

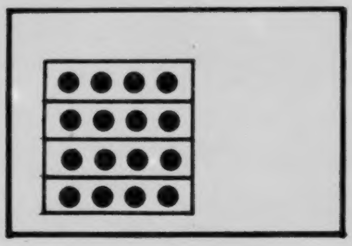
Pupil's book, page 382: This page should be discussed with the children.

Pupil's book, pages 383 - 384: These pages are for independent work. Problems on page 383 may have to be read aloud with some groups, however.

Using Multiplication



Picture A



Picture B

Look at Picture A.

How many cards of buttons do you see? 4

How many buttons are on each card? 4

Look at Picture B.

The cards of buttons have been put together.

What multiplication equation tells you how many buttons there are?

4 x 4 = 16

Using Multiplication

Draw a line to the equation that is related to each problem.

Tom had 5 coins on each card.
He has 7 cards of coins.
How many coins did he have?

Mary had 3 boxes of pencils.
There were 6 pencils in each box.
How many pencils did she have?

Karen can put 4 flowers in each vase.
All together she had 28 flowers.
How many vases does she need?

Tom wants to give each of his friends 5 marbles.
He needs 35 marbles.
How many friends will get marbles?

Bill went on a 7 day vacation.
He caught the same number of fish each day.
That week he caught 42 fish.
How many fish did he catch a day?

$7 \times 42 = n$

$3 \times 6 = n$

$7 \times 5 = n$

$6 \times 3 = n$

$n \times 7 = 42$

$n \times 5 = 35$

$n \times 4 = 28$

$5 \times 35 = n$

$7 \times n = 42$

$4 \times 28 = n$

$5 \times 7 = n$

Using Multiplication

Complete. Your multiplication chart should help you.

3 × 5 = 15

5 × 6 = 30

2 × 9 = 18

3 × 6 = 18

7 × 4 = 28

3 × 8 = 24

4 × 2 = 8

5 × 10 = 50

7 × 10 = 70

100 × 8 = 800

100 × 6 = 600

3 × 4 = 12

2 × 3 = 6

5 × 2 = 10

3 × 6 = 18

5 × 4 = 20

3 × 7 = 21

9 × 1 = 9

2 × 10 = 20

1 × 100 = 100

100 × 5 = 500

100 × 9 = 900

VII-3. Division

Objective: To introduce the idea of division.

Vocabulary: Divide, division, quotient, and the symbol ÷ .

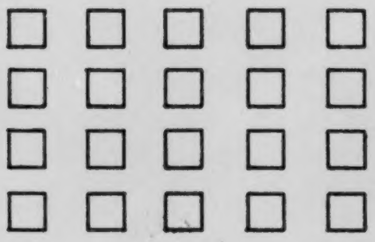
Materials: 23 cards (baseball cards or pieces of paper cut out to look like them), pennies, other manipulative materials.

Suggested Procedure:

Propose the problem:

> Jimmy has a collection of baseball cards. When he looked through them he found that he had 20 he didn't want because he had some just like them. He decided to sell the 20 cards in packages of 5. How many packages of 5 could he make from his 20 cards? In our last lesson we learned to write a multiplication equation for a problem like this. What equation should we write? ($n \times 5 = 20$.) What multiplication fact helps us answer this problem? Today we are going to learn how we can find the answer to problems like this in another way.

Ask the class how Jimmy might find out. Listen for the suggestion to count them out in fives. Do this, using five cards in each row and forming a 4 by 5 array:



Say that when Jimmy divides up his extra cards, he gets 4 sets of 5 cards. Explain that we can write a division equation to show what happened: $\frac{20}{5} = 4$. Tell how it is read: twenty divided by five is equal to four. Explain that when we divide one number by another, the result is called the quotient. In the equation, 20 divided by 5 is equal to four, four is the quotient. Also relate the statement to the problem situation--20 in the equation to the 20 cards in the problem, 5 in the equation to the 5 cards in each package in the problem, and 4 in the equation to the 4 packages of 5 in the problem.

Since children should be familiar with both symbols for division (the bar which was previously introduced in Grades 1 and 2 and ÷), the other symbol (÷) should be introduced at this time.

Explain to the children that there is another way the division equation may be written: $20 \div 5 = 4$. Point out that $\frac{20}{5} = 4$ and $20 \div 5 = 4$ are read in exactly the same way. Relate the story problem to the new form of the equation using the procedure suggested above. Make sure in future work that practice in using both forms of the division equation is provided.

Then vary the problem situation so that Jimmy has a different number of cards to give away. Select numbers so that the division is complete, i.e. no remainder. For example,

> Suppose Jimmy had 45 cards.
> He still wanted to put them in packages
> of 5.
> How many packages will he have?

Continue with other examples not only varying the number of cards Jimmy gives away, but also the number of cards he puts in each package. Or, if you wish to change the problem situation to separating something else into equivalent subsets, do so. Find the answer to each question by using manipulative materials. After each solution has been found, have the children decide which equation they can write.

Use sufficient numbers of problem situations to develop the idea of separating a set of objects into equivalent subsets and then relate the appropriate equation etc., to the action that has taken place.

Although pairs of numbers need not be restricted to the basic facts, it seems more appropriate to do so. Some of the pairs might be the following: 32 and 8, 35 and 7, 24 and 8, 36 and 6, etc., where the first member is the number of objects in the set and the second the number of objects to be placed in each equivalent subset.

Although it is easier in a problem situation to consider the division as a number that indicates the number of members in a row of the array, children need not do so. For example, $\frac{10}{2} = 5$ or $10 \div 5 = 2$ can be shown as either a 2×5 or a 5×2 array.

Pupil's book, pages 385 - 386: Children may ring sets to show partitions (For $8 \div 4$, they would ring sets of 4.), draw arrays to show a rearrangement of the objects in the set to be partitioned, or use manipulative materials to form such arrays. Provide help as necessary.

Pupil's book, page 387: Children should use manipulative materials or draw arrays as needed.

Learning About Division

Finish these. Show an array for each problem.

(a) $8 \div 4 =$ 2

(b) $\frac{12}{4} =$ 3

(c) $15 \div 5 =$ 3

(d) $\frac{16}{4} =$ 4

(e) $32 \div 8 =$ 4

(f) $\frac{30}{6} =$ 5

Division.

Finish these. Show an array for each problem.

(g) $21 \div 7 =$ 3

(h) $27 \div 3 =$ 9

(i) $\frac{64}{8} =$ 8

(j) $\frac{81}{9} =$ 9

(k) $18 \div$ 6 $= 3$

(l) $\frac{24}{4} = 6$

(m) 20 $\div 5 = 4$

(n) $\frac{30}{6} = 5$

Division

Use arrays to help you if you need them.

$10 \div 5 = 2$

$12 \div 3 = 4$

$16 \div 4 = 4$

$40 \div 5 = 8$

$32 \div 4 = 8$

$24 \div 3 = 8$

$45 \div 5 = 9$

$21 \div 3 = 7$

$12 \div 6 = 2$

$18 \div 3 = 6$

$56 \div 7 = 8$

$12 \div 4 = 3$

$27 \div 3 = 9$

$16 \div 8 = 2$

$54 \div 6 = 9$

$35 \div 7 = 5$

$28 \div 4 = 7$

$36 \div 4 = 9$

$49 \div 7 = 7$

$48 \div 8 = 6$

$81 \div 9 = 9$

$42 \div 6 = 7$

VII-4 Quotients

Objective: To develop a technique for finding a quotient.

Vocabulary: (No new words.)

Materials: (No special materials.)

Suggested Procedure:

Present the following problem, or a similar one:

> Jimmy was waiting for a dental appointment, and he began thinking about 40 baseball cards he would like to sell. He wondered how many sets of 5 he could make of the 40 cards. Since he didn't have the cards with him, he couldn't actually arrange them in sets of 5, and he couldn't remember any of the multiplication facts for 5 except $1 \times 5 = 5$, $2 \times 5 = 10$, $3 \times 5 = 15$, $4 \times 5 = 20$, and $5 \times 5 = 25$.

Write the multiplication facts mentioned above and the problem Jimmy wanted to solve, $\frac{40}{5} = \_\_\_$, on the chalkboard. Explain that Jimmy borrowed a pencil and paper from the woman in the office and found out how many sets of the facts he knew, without drawing an array. Ask children to suggest ways he might have done this. Allow children to explore any possibilities which fit the conditions of the problem: no drawings and no multiplication facts beyond $5 \times 5 = 25$.

It is possible that the children, themselves, may provide an introduction to the division algorithm. If not, you may suggest that Jimmy would know he could make at least 5 sets of 5 cards, because $5 \times 5 = 25$, (write $5 \times 5 = 25$ on the chalkboard) and 25 is less than 40. If he made 5 sets, he would have $40 - 25$ or 15 cards left. (Write $40 - 25 = 15$.) From the 15 cards he could make 3 more sets of 5, because $3 \times 5 = 15$. (Write $3 \times 5 = 15$.) All together he could make 8 sets, because $5 + 3 = 8$. (Write $5 + 3 = 8$.)

In your own words, point out the fact that you have written $\frac{40}{5} = \_\_\_$, $5 \times 5 = 25$, $40 - 25 = 15$, $3 \times 5 = 15$, and $5 + 3 = 8$ in trying to solve the problem

Introduce the division algorithm by suggesting that it would be convenient to write the 25 under the 40 in order to subtract, and that this can be done if the numeral 5 is moved over, out of the way. Show:

$$5\,\overline{)40}$$

Ask what fact Jimmy used first. ($5 \times 5 = 25$.) Explain that you will write the 5 to the right, so you will remember which fact you used first, and will write the product of 5 and 5 under the 40 in order to subtract and find out what number is still to be divided.

$$\begin{array}{r|l} 5\,\overline{)40} & \\ \underline{25} & 5 \\ 15 & \end{array}$$

Explain again that Jimmy had found he could make 5 sets of 5 cards, but he knew he had 15 of his 40 cards left. What fact did he use then? ($3 \times 5 = 15$.) Show how to record the 3 under the 5 on the right and how to subtract the product of 3 and 5 from 15.

$$\begin{array}{r|l} 5\,\overline{)40} & \\ \underline{25} & 5 \\ 15 & \\ \underline{15} & 3 \end{array}$$

What did Jimmy do to find out how many sets he can make all together? (Added. $5 + 3 = 8$).

$$\begin{array}{r|r|l} 5 & 40 & \\ & 25 & 5 \\ \cline{2-3} & 15 & \\ & 15 & 3 \\ \cline{2-3} & & 8 \end{array}$$

Suggest that if Jimmy had known that $6 \times 5 = 30$ he might have solved the problem differently:

$$\begin{array}{r|r|l} 5 & 40 & \\ & 30 & 6 \\ \cline{2-3} & 10 & \\ & 10 & 2 \\ \cline{2-3} & & 8 \end{array}$$

However, he would have arrived at the same quotient. Repeat the procedure with the idea that Jimmy knew $7 \times 5 = 35$, $8 \times 5 = 40$, and $9 \times 5 = 45$. Emphasize the fact that using $8 \times 5 = 40$ would have eliminated much pencil work:

$$\begin{array}{r|r|l} 5 & 40 & \\ & 40 & 8 \\ \cline{2-3} & & \end{array}$$

Jimmy would not have used $9 \times 5 = 45$, because he knew he could not make 9 sets of 5 from only 40 cards.

Continue with many other examples: $\frac{36}{4}$, $63 \div 7$, $56 \div 8$, etc., to help children learn how they can find quotients even though they may not know the immediately appropriate multiplication facts. In work with division, however, permit those children who need to use manipulative materials or to draw arrays to do so.

Pupil's book, pages 388-391: The first of these pages should be discussed with the children. The others are for independent work.

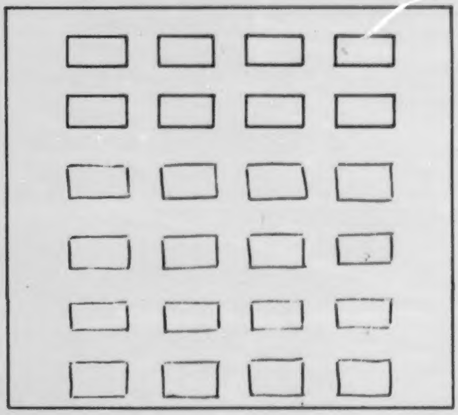
Learning to Divide

Bill had 24 tickets.

He put them in sets of 4.

How many sets of tickets did he have?

Finish this array to show how Bill could have separated the tickets.



How many sets of 4 would he have? 6

Now can you find the number of sets without the tickets?

24 ÷ 4 = 6

$$\begin{array}{r|l} 4\overline{)\,24} & \\ 16 & 4 \\ \hline 8 & \end{array}$$

4 sets of tickets will take 16 tickets.

There are 8 tickets left.

How many sets can he make now? 2

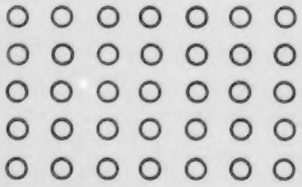
How many sets of tickets has he in all? 6

Are there any tickets left over? 0

Division

Complete: Use symbols or pictures. The first has been done for you using both.

1. $35 \div 7 =$ 5



$35 \div 7 = 5$

$$\begin{array}{r|l} 7\overline{)\,35} & \\ \underline{21} & 3 \\ 14 & \\ \underline{14} & \underline{2} \\ & 5 \end{array}$$

$35 \div 7 = 5$

2. $\frac{42}{6} =$ 7

3. $\frac{32}{8} =$ 4

4. $54 \div 9 =$ 6

5. $\frac{24}{3}$ = 8

6. 36 ÷ 4 = 9

7. $\frac{42}{7}$ = 6

8. 64 ÷ 8 = 8

9. $\frac{27}{3} =$ 9

10. 48 ÷ 8 = 6

11. $\frac{63}{7} =$ 9

12. 49 ÷ 7 = 7

VII-5. Relating division to multiplication

Objective: To relate division to multiplication.

Vocabulary: (No new words.)

Materials: Manipulative materials.

Teaching Comment:

In developing the ideas in this section, we want to help children to see the relationship between division and multiplication, that $12 \div 3 = 4$ and $4 \times 3 = 12$. More specifically to find the quotient, for two numbers, they can think what number times 3 is equal to 12, or $n \times 3 = 12$. Although they have been using this idea in the previous section, we now give specific attention to it.

Suggested Procedure:

Recall some of the problem situations that have been used earlier with division or suggest a new problem. For example, returning to the problems about Jimmy and his baseball cards,

> Let's think about Jimmy and those baseball cards.
> Suppose Jimmy had 20 cards.
> He put them in sets of 4.
> How many sets would he have?

Have someone show the result using a set of cards. Then ask someone to write the division equations. ($20 \div 4 = n$ or $\frac{20}{4} = n$.)

Now ask another child to put these equivalent subsets together to form an array. Ask what multiplication equation tells us about the number of objects in the array. Ask a child to write this equation. ($n \times 4 = 20$ or $4 \times n = 20$.)

Let's look at these two equations.

$$20 \div 4 = 5$$
$$4 \times 5 = 20$$

What number did we find in the first equation?
What number did we find in the second equation?

Discuss these equations. In the first equation we divided by 4. In the second equation we multiplied by 4. We started with 20. We ended with 20.

Use one or two more problem situations where the children first separate the set into equivalent subsets. They use these subsets to make an array. For each problem, have them write the two equations. Discuss these in much the same way as the one above.

Then start with a multiplication problem.

Suppose there are 6 chairs in a row.
(Can use objects on flannelboard to represent chairs.)
Suppose we have 3 rows of chairs.
How many chairs do we have altogether?

Then have them separate the 3 rows of 6 chairs each.

Again ask for the equations:

Thinking of the chairs together: $3 \times 6 = 18$.
Separating the chairs into rows: $18 \div 3 = 6$.

Here they see that multiplying six by three and then dividing that result by three results in the number six. Continue with other examples to help children see that to multiply by a number and then to divide by that same number results in the number with which we began.

Also, to divide by a number and then to multiply by the same number results in the number with which we started.

Put examples such as the following on the chalkboard:

3 × 4 = 12 Ask for the division equation that undoes multiplying by 3.

(12 ÷ 3 = 4.)

8 × 2 = 16 and 16 ÷ 8 = 2
6 × 4 = 24 and 24 ÷ 6 = 4
5 × 5 = 25 and 25 ÷ 5 = 5
4 × 9 = 36 and 36 ÷ 4 = 9

Pupil's book, pages 392 - 393: deal with doing and undoing.

page 394 is concerned with writing multiplication equations.

page 395 requires that the children write division equations.

Relation of Multiplication and Division

A. Multiplying by a number will undo dividing by the same number.

Think of 8.
Divide 8 by 2. $8 \div 2 = 4$
Then multiply 4 by 2. $2 \times 4 = 8$
The result is 8, the original number.
Multiplying by 2 undid dividing by 2.

B. Dividing by a number will undo multiplying by the same number.

Think of 8.
Multiply by 2. $2 \times 8 = 16$
Then divide 16 by 2. $16 \div 2 = 8$
The result is 8, the original number.

C. Fill the blanks to show doing and undoing.

The first example is done for you.

DO	UNDO
2 × 3 = 6	6 ÷ 2 = 3
4 × 5 = 20	20 ÷ 4 = 5
6 × 4 = 24	24 ÷ 4 = 6
3 × 7 = 21	21 ÷ 7 = 3
8 × 4 = 32	32 ÷ 4 = 8
10 × 4 = 40	40 ÷ 4 = 10
7 × 8 = 56	56 ÷ 8 = 7
9 × 3 = 27	27 ÷ 3 = 9
4 × 3 = 12	12 ÷ 3 = 4
18 ÷ 3 = 6	3 × 6 = 18
64 ÷ 8 = 8	8 × 8 = 64
16 ÷ 4 = 4	4 X 4 = 16
42 ÷ 7 = 6	6 X 7 = 42
81 ÷ 9 = 9	9 X 9 = 81
24 ÷ 3 = 8	8 X 3 = 24
45 ÷ 5 = 9	5 X 9 = 45
36 ÷ 6 = 6	6 X 6 = 36

Multiplication Equations

Think of 45 ÷ n = 9 as n × 9 = 45.

Write the multiplication equation.

The first one has been done for you.

21 ÷ n = 7 n × 7 = 21 n = 3	16 ÷ n = 4 n x 4 = 16 n = 4	24 ÷ n = 4 n x 4 = 24 n = 6
40 ÷ n = 8 n x 8 = 40 n = 5	30 ÷ n = 6 n x 6 = 30 n = 5	25 ÷ n = 5 n x 5 = 25 n = 5
24 ÷ n = 8 n x 8 = 24 n = 3	16 ÷ n = 2 n x 2 = 16 n = 8	15 ÷ n = 5 n x 5 = 15 n = 3
28 ÷ n = 4 n x 4 = 28 n = 7	27 ÷ n = 9 n x 9 = 27 n = 3	12 ÷ n = 2 n x 2 = 12 n = 6
35 ÷ n = 7 n x 7 = 35 n = 5	20 ÷ n = 5 n x 5 = 20 n = 4	32 ÷ n = 8 n x 8 = 32 n = 4

Division Equations

Think of n × 5 = 45 as 45 ÷ 5 = 9 .

Write the division equation.

The first one has been done for you.

n × 4 = 24 24 ÷ 4 = 6	n × 7 = 21 21 ÷ 7 = 3	n × 9 = 45 45 ÷ 9 = 5
n × 6 = 18 18 ÷ 6 = 3	n × 7 = 42 42 ÷ 7 = 6	n × 9 = 63 63 ÷ 9 = 7
n × 8 = 56 56 ÷ 8 = 7	n × 9 = 72 72 ÷ 9 = 8	n × 9 = 27 27 ÷ 9 = 3
n × 7 = 21 21 ÷ 7 = 3	n × 3 = 24 24 ÷ 3 = 8	n × 7 = 49 49 ÷ 7 = 7
n × 9 = 36 36 ÷ 9 = 4	n × 9 = 18 18 ÷ 9 = 2	n × 4 = 16 16 ÷ 4 = 4
n × 6 = 36 36 ÷ 6 = 6	n × 9 = 54 54 ÷ 9 = 6	n × 8 = 48 48 ÷ 8 = 6
n × 8 = 40 40 ÷ 8 = 5	n × 8 = 64 64 ÷ 8 = 8	n × 9 = 81 81 ÷ 9 = 9

VII-6. The associative property of multiplication

Objective: To introduce the idea of the associative property of multiplication.

Vocabulary: Associative property of multiplication.

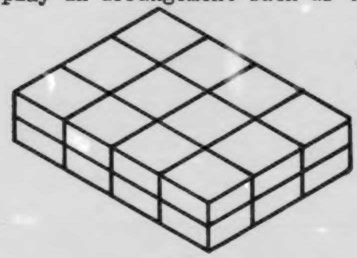
Materials: Blocks, sugar cubes, rectangular prisms.

Suggested Procedure:

On the flannel board or table, place three sets of objects. Ask children what mathematical equation is suggested by joining the three sets. For example, suppose the sets had 3, 2, and 4 numbers. The equation $3 + 2 + 4 = 9$. Ask children what property of addition is used to add three numbers. (Associative or grouping) Have the children show it doesn't make any difference in the sum which 2 numbers are added first. Review the idea of associative property.

$$(3 + 2) + 4 = 3 + (2 + 4)$$
$$5 + 4 = 3 + 6$$
$$9 = 9$$

Now display an arrangement such as the following.



Ask if anyone can think of an equation that is suggested by this arrangement. If children do not respond, direct their attention to the "top layer"--the number of rows and the number of objects in each row. Then ask how they could describe the number of arrays that they had.

For example, (3 × 4) one layer

(2 × 3 × 4) both layers

Help children the see the 3 by 4 array and that the product of 3 and 4 multiplied by 2 gives one result. Write 2 × (3 × 4) = 24.

Then ask if they would get the same results if they found the product of 2 and 3 first. Ask what this would mean using the model. (Find number in array along one side. Then find how many in four such arrays.)

Work the two examples on the chalkboard.

$$2 \times (3 \times 4) = (2 \times 3) \times 4$$
$$2 \times 12 = 6 \times 4$$
$$24 = 24$$

Tell the children that we have used another mathematical property. Ask them if they know what property if is. (Associative or grouping property of multiplication.)

If you use additional equations, be careful to select them so that none requires facts beyond 5 × 9 and 9 × 5.

$$(2 \times 2) \times 3 = 2 \times (2 \times 3)$$
$$(3 \times 2) \times 2 = 3 \times (2 \times 2)$$
$$(2 \times 1) \times 5 = 2 \times (1 \times 5)$$
$$2 \times (3 \times 3) = (2 \times 3) \times 3$$

Explain that sometimes the associative law helps us to solve problems which we would otherwise not have had enough facts to solve.

Write: (4 × 3) × 2

What is 4 × 3? (12.)

Now we must find the product of 12 and 2 but we haven't yet learned that multiplication fact. How can we find the product?

We can group the factors in another way.

Write: $4 \times (3 \times 2)$ and ask whether this is the same number as $(4 \times 3) \times 2$. (Yes.) Have a child give the product of 3×2. Write: 4×6. Ask another child what the product of 4×6 is. Write: $4 \times 6 = 24$. Be sure your record looks like that below so children can see that in solving one equation they have the solution to the other.

$$\begin{aligned} n &= 4 \times (3 \times 2) \\ &= 4 \times 6 \\ &= 24 \end{aligned}$$

In the same way, present several of the equations below. Have them rewritten in such a form that the children can find the products with the facts at their command.

$$\begin{aligned} n &= 2 \times (4 \times 5) \\ &= 2 \times 20 \\ &= 40 \end{aligned} \qquad \begin{aligned} n &= (2 \times 4) \times 5 \\ &= 8 \times 5 \\ &= 40 \end{aligned}$$

Other practice equations:

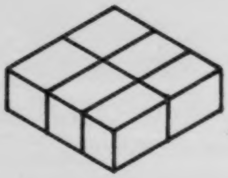
$3 \times 3 \times 4$ $\qquad$ $3 \times 3 \times 5$

$2 \times 2 \times 5$ $\qquad$ $3 \times 2 \times 5$

Pupil's book, pages 396 - 399: It may be necessary to do these pages as a class discussion. If some children can do them on their own, they should be encouraged to do so.

Multiplying Three Numbers

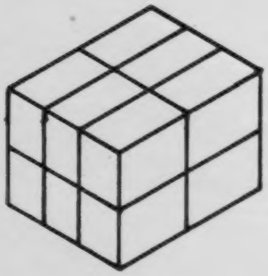
1.



Here is a picture of a set of blocks, arranged in 2 rows of 3 blocks. How many blocks are used? 6

2 × 3 = 6

2.



Here is a picture of a set of blocks arranged in 2 rows of 3 blocks and 2 layers. How many blocks were used in one layer? 6

2 × 3 = 6

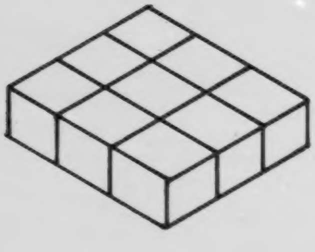
How many blocks were used in 2 layers?

n = (2 × 3) × 2

= 6 × 2

= 12

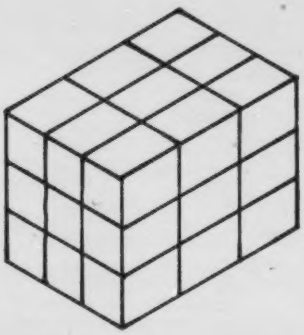
3.



Here is a picture of a set of blocks with 3 rows and 3 blocks in each row. There are 9 blocks in the picture.

3 × 3 = 9

4.



Here is a picture of a set of blocks with 3 rows of 3 blocks and 3 layers. How many blocks were used?

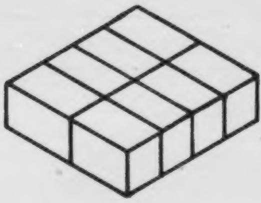
$n = 3 \times 3 \times 3$

$= (3 \times 3) \times 3$

$= 9 \times 3$

$= \underline{\ 27\ }$

5.

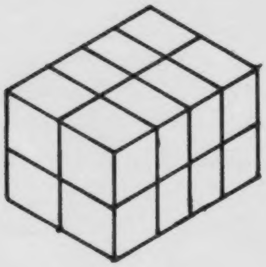


Here is a picture of a set of blocks with 2 rows and 4 blocks in each row. The equation that tells us the number of blocks is:

$\underline{\ 2\ } \times \underline{\ 4\ } = \underline{\ 8\ }$.

6.

There are 2 layers of blocks. In each layer there are 2 rows of 4 blocks. In the picture there are 16 blocks.



$n = 2 \times (2 \times 4)$

$= 2 \times \underline{\ 8\ }$

$= \underline{\ 16\ }$

What would you find for the product if you write:

$(2 \times 2) \times 4 = \underline{\ 4\ } \times \underline{\ 4\ } = \underline{\ 16\ }$

Are your products the same? Yes.

Multiplication

Example: 2 × 3 × 4 = (2 × 3) × 4
= 6 × 4
= 24
OR
2 × 3 × 4 = 2 × (3 × 4)
= 2 × 12
= 24

First multiply the factors in parentheses ( ).

Did you get the same product? Yes

Why? Associative property of multiplication.

Find these products:

1. 3 × 3 × 2

3 × 3 × 2 = (3 × 3) × 2
= 9 × 2
= 18

3 × 3 × 2 = 3 × (3 × 2)
= 3 × 6
= 18

2. 2 × 4 × 5

2 × 4 × 5 = (2 × 4) × 5
= 8 × 5
= 40

2 × 4 × 5 = 2 × (4 × 5)
= 2 × 20
= 40

3. Find the product of 4, 2, and 3 in two ways.

4 × 2 × 3 = (4 × 2) × 3
= 8 × 3
= 24

4 × 2 × 3 = 4 × (2 × 3)
= 4 × 6
= 24

4. Find the product of 2, 4, and 5 in two ways.

2 × 4 × 5 = (2 × 4) × 5
= 8 × 5
= 40

2 × 4 × 5 = 2 × (4 × 5)
= 2 × 20
= 40

5. Find the product of 5, 2, and 3 in two ways.

5 × 2 × 3 = (5 × 2) × 3
= 10 × 3
= 30

5 × 2 × 3 = 5 × (2 × 3)
= 5 × 6
= 30

6. Find the product of 2, 5, and 4 in two ways.

2 × 5 × 4 = (2 × 5) × 4
= 10 × 4
= 40

2 × 5 × 4 = 2 × (5 × 4)
= 2 × 20
= 40

VII-7. Multiplying and dividing 10 and multiples of 10

Objective: To use the associative property in multiplication with multiples of ten in both multiplication and division.

Vocabulary: (No new words.)

Materials: Manipulative materials, array of objects.

Suggested Procedure:

Review thinking of multiples of ten in more than one way by asking how to rename 70 as tens. (7 tens.) Continue with many other multiples: 200, 360, 290, 140, 70, 250, 450, 100, etc.

Ask for another way to think about 3 tens (30). Ask if someone can make an array showing 3 tens. (3 rows of 10 members each.) This may be drawn on the chalkboard or arranged with objects. Ask what multiplication equation could be made for the array. (3 × 10 = 30.) State that 3 × 10 is another way of saying 3 tens. Ask what 4 × 10 is, and if the answer is 4 tens, have the number renamed as 40. Continue with 7 × 10, 2 × 10, 8 × 10, etc. Ask what 12 × 10 is. If there is hesitation, remind children that this is 12 tens, and therefore 120. Give much practice with multiplying a number times 10, including numbers such as 45 × 10.

Continue practice, including examples like 24 × 10 and 10 × 9, 7 × 10 and 10 × 25. Refer to 8 and 10 as factors of 80.

On the chalkboard write 3 × 40. Ask how 40 can be written using multiplication of 2 numbers. (4 × 10.) Then write 3 × (4 × 10) and ask whether this means 3 × 4. (Yes.)

The written records shows: $3 \times 40 = 3 \times (4 \times 10)$

Ask how the associative property can be used to rewrite $3 \times (4 \times 10)$.

Write: $= (3 \times 4) \times 10$

Ask a child for the product of 3×4.

Write: $= 12 \times 10$

Ask another child for the product of 12×10.

Write: $= 120$

Follow the same procedure using 7×200. Thus:

$$\begin{aligned} 7 \times 200 &= 7 \times (2 \times 100) \\ &= (7 \times 2) \times 100 \\ &= 14 \times 100 \\ &= 1400 \end{aligned}$$

Provide practice on such equations as $4 \times 60 = 240$, $2 \times 90 = 180$, $9 \times 30 = 270$, $7 \times 40 = 280$, etc.

Discourage any suggestion of "just adding a zero" to the number. If this arises, show that if a zero is added to 12 in the illustrative equation, for example the sum of 12 and 0 is 12. If, however, children do not show all the steps in solving the equations on the pupil pages, allow them to continue to do the work mentally.

For those pupils who encounter no difficulty with multiplying using multiples of 10, use a similar procedure to show the use of multiples of 100: $300 = 3$ hundreds $= 3 \times 100$; $4 \times 600 = 4 \times (6 \times 100) = (4 \times 6) \times 100 = 24 \times 100 = 2400$; etc.

Pupil's book, pages 400 - 403: These pages provide practice.

Multiplying with Multiples of 10

Multiply:

45 × 10 = 450
10 × 35 = 350
72 × 10 = 720
12 × 10 = 120
7 × 10 = 70
9 × 10 = 90
10 × 50 = 500
10 × 39 = 390
4 × 100 = 400
6 × 200 = 1200
3 × 600 = 1800
6 × 10 = 60
10 × 84 = 840
3 × 10 = 30
24 × 10 = 240
18 × 10 = 180
37 × 10 = 370

17 × 10 = 170
10 × 72 = 720
100 × 7 = 700
5 × 800 = 4000
400 × 3 = 1200
27 × 10 = 270
64 × 10 = 640
10 × 53 = 530
10 × 47 = 470
68 × 10 = 680
6 × 10 = 60
8 × 10 = 80
10 × 11 = 110
2 × 500 = 1000
7 × 400 = 2800
100 × 8 = 800
50 × 20 = 1000

A Chart

Directions: Fill in the chart with the products for the numbers in the rows and the columns.

Example: 8 × 30 = 240

You may think about 8 × 30 = 8 × (3 × 10)
= (8 × 3) × 10
= 24 × 10
= 240

×	10	20	30	50	90	80	60	70	40
1	10	20	30	50	90	80	60	70	40
2	20	40	60	100	180	160	120	140	80
3	30	60	90	150	270	240	180	210	120
5	50	100	150	250	450	400	300	350	200
8	80	160	240	400	720	640	480	560	320
4	40	80	120	200	360	320	240	280	160
6	60	120	180	300	540	480	360	420	240
9	90	180	270	450	810	720	540	630	360
7	70	140	210	350	630	560	420	490	420

Using the Associative Property for Multiplication

Multiply:

1) 3 × 20 = 3 × (2 × 10)
= (3 × 2) × 10
= 6 × 10
= 60

2) 5 × 70 = 5 × (7 × 10)
= (5 × 7) × 10
= 35 × 10
= 350

3) 4 × 20 = 80

4) 3 × 40 = 120

5) 5 × 90 = 450

6) 7 × 40 = 280

7) 70 × 3 = 10 × 7 × 3

= 10 × (7 × 3)

= 10 × 21

= 210

8) 50 × 9 = (10 × 5) × 9

= 10 × (5 × 9)

= 10 × 45

= 450

9) 60 × 4 = 240

10) 80 × 2 = 160

11) 600 × 3 = (100 × 6) × 3

= 100 × (6 × 3)

= 100 × 18

= 1800

12) 900 × 4 = 3600

13) 400 × 8 = 3200

14) 300 × 6 = 1800

Dividing multiples of ten

Review renaming 20 as 2 tens, and as 2 × 10. After giving several examples, state that any number which is the product of ten and another factor is a multiple of 10, including numbers like 100, where the other factor is also 10. Have many multiples of 10 named and described as $n \times 10$ and as $10 \times n$.

Write $70 \div 10$. If children hesitate, show the equation $n \times 10 = 70$. Continue to give examples including problems where 10 is the quotient as well as problems in which 10 is the divisor. (It is not necessary to use the term "divisor" with the children at this time.) Use multiples in which one factor is greater than the other as in $450 \div 10$ or $210 \div 7$. Also use multiples of 100 if examples such as 8×100 and 100×5 have been given previously.

Write $120 \div 3$. Rewrite immediately as $n \times 3 = 120$. Ask what multiple of 10 would be needed to solve the equation. If children hesitate, remind them that 120 is 12 tens.

Give much practice in class before using pupil page. Examples: $70 \div 10$, $140 \div 7$, $\frac{250}{10}$, etc.

Pupil's book, page 404: This page provides practice in in division.

Multiples of Ten

70 ÷ 7 = 10

70 ÷ 10 = 7

140 ÷ 7 = 20

60 ÷ 6 = 10

120 ÷ 6 = 20

60 ÷ 10 = 6

20 ÷ 2 = 10

40 ÷ 2 = 20

60 ÷ 2 = 30

20 ÷ 10 = 2

40 ÷ 10 = 4

$\frac{80}{8}$ = 10

$\frac{80}{10}$ = 8

$\frac{160}{8}$ = 20

$\frac{50}{5}$ = 10

$\frac{100}{5}$ = 20

$\frac{50}{10}$ = 5

$\frac{20}{10}$ = 2

$\frac{30}{10}$ = 3

$\frac{40}{4}$ = 10

$\frac{150}{15}$ = 15

$\frac{100}{10}$ = 10

40 ÷ 10 = 4

80 ÷ 4 = 20

90 ÷ 9 = 10

$\frac{180}{9}$ = 20

90 ÷ 9 = 10

$\frac{30}{3}$ = 10

130 ÷ 10 = 13

120 ÷ 10 = 12

$\frac{160}{10}$ = 16

140 ÷ 10 = 14

190 ÷ 10 = 19

VII-8. The distributive property

Objective: To help children understand the distributive property of multiplication over addition as illustrated by $a \times (b + c) = (a \times b) + (a \times c)$; e.g., $3 \times 14 = 3 \times (10 + 4) = (3 \times 10) + (3 \times 4)$.

Vocabulary: (No new words.)

Materials: Arrays, squared paper, yarn.

Suggested Procedure:

Use a 3 by 12 array. Ask children how they can find out how many objects are in the array. They might suggest using addition--$12 + 12 + 12 = 36$. Also, some may suggest counting. Then tell them that today they are going to learn another way.

Fold or cut the 3 by 12 array so that you have two smaller arrays. Describe each of the smaller arrays. For example, you may separate the array so as to have a 3 by 10 array and a 3 by 2 array.

We can describe the array as 3×12 or $3 \times (10 + 2)$.

So write

$$\begin{aligned} 3 \times 12 &= 3 \times (10 + 2) \\ &= (3 \times 10) + (3 \times 2) \end{aligned}$$

(Three rows of 12 objects separated into 3 rows of 10 objects and 3 rows of 2 objects.)

Ask someone to give the product of 3×10, of 3×2. Continue with the writing

$$3 \times 12 = 30 + 6.$$

Then ask for another way of naming the number $(30 + 6)$. (36.) Write: $30 + 6 = 36$.

Consider other separations of the 3 by 12 array into 2 arrays. For example,

$$\begin{aligned} 3 \times 12 &= 3 \times (6 + 6) \\ &= (3 \times 6) + (3 \times 6) \\ &= 18 + 18 \\ &= 36 \end{aligned}$$

$$\begin{aligned} 3 \times 12 &= 3 \times (4 + 4 + 4) \\ &= (3 \times 4) + (3 \times 4) + (3 \times 4) \\ &= 12 + 12 + 12 \\ &= 36 \end{aligned}$$

etc.

Observe the different ways used. Ask if any one way seemed easier than the others. Hopefully we would expect the first example, $3 \times 12 = 3 \times (10 + 2)$. However, children may not feel familiar with products of 10 to think that this is easier. If so, you should give further opportunity to give the products when 10 is one of the factors, such as,

3×10
2×10
4×10

10×5
10×2
10×4

as was done in the preceding sections. This may be followed by practice in renaming numbers such as $24 = 20 + 4$, $32 = 30 + 2$, $47 = 40 + 7$, etc.

Show the children that there is an easier form they may use.

$$\begin{aligned} 3 \times 12 &= 3 \times (10 + 2) \\ &= (3 \times 10) + (3 \times 2) \\ &= 30 + 6 \\ &= 36 \end{aligned}$$

Write: $$\begin{array}{r} 10 + 2 \\ \underline{\times\ 3} \\ 30 + 6 = 36 \end{array}$$

Continue with several other examples

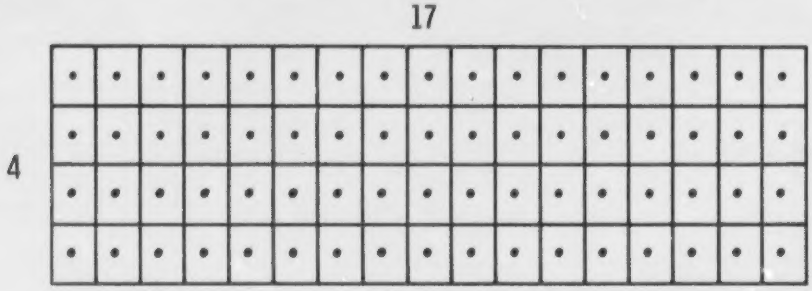
3×21 $$\begin{array}{r} 20 + 1 \\ \underline{\times\ 3} \\ 60 + 3 = 63 \end{array}$$

4×32 $$\begin{array}{r} 30 + 2 \\ \underline{\times\ 4} \\ 120 + 8 = 128 \end{array}$$

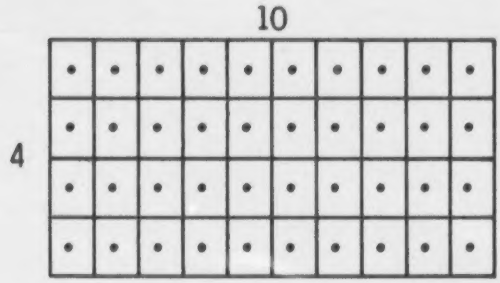
Pupil's book, pages 405 - 408: These pages provide practice using the distributive property.

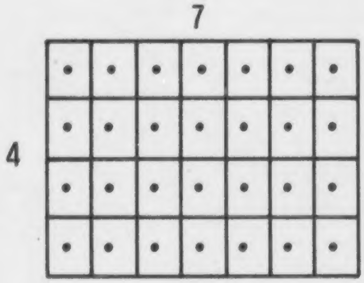
The Distributive Property

1. To find the product for $4 \times 17 = n$, think of a 4 by 17 array.



Then think about separating this array into two smaller arrays.





Find the number of members in each array.

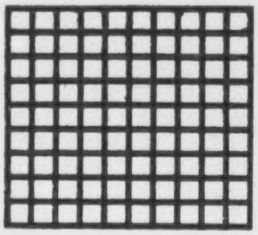
Then add these numbers to find the number of members in the 4×17 array.

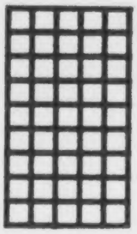
We write:

$$\begin{array}{r} 17 \\ \times\ \underline{4} \end{array} \qquad\qquad \begin{array}{r} 10 + 7 \\ \times\ 4 \\ \hline 40 + 28 = 68 \end{array}$$

The number of members in a 9 by 10 array is 9 × 10 = 90

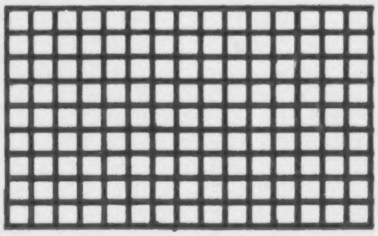
The number of members in a 9 by 5 array is 9 × 5 = 45





The number of members in a 9 by 15 array is 9 × 15 = 135

$$\begin{array}{r} 15 \\ \times 9 \\ \hline \end{array} \qquad \begin{array}{r} 10 + 5 \\ \times 9 \\ \hline 90 + 45 = 135 \end{array}$$

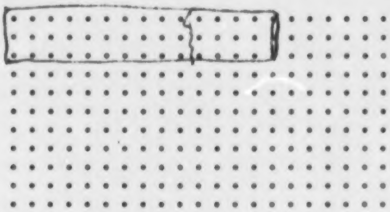


Using Arrays

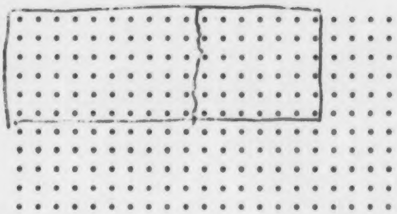
Find the products:

Show the array:

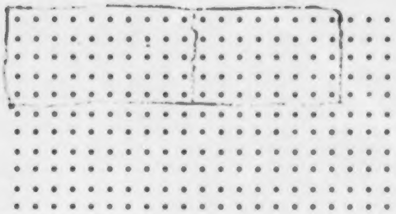
1. 3 × 15 = 45



2. 6 × 17 = 102



3. 5 × 18 = 90



Multiplication

Try these. Use arrays to help you if you need them.

1. 17 ×2 34	2. 15 ×4 60
3. 18 ×3 54	4. 17 ×6 102
5. 19 ×5 95	6. 13 ×9 117
7. 12 ×4 48	8. 17 ×3 51

VII-9. Solving problems

The following Pupil's pages are included to provide additional experiences in solving story problems.

It is not intended that every child must complete every page in consecutive order. The teacher's own judgment should be the guide. Independent work is to be encouraged whenever feasible.

Less able children should be encouraged to use manipulative materials whenever they need them. Very able children might explain the solutions for difficult problems to the class.

Problem Set 1,	Pupil's book, page 409
Problem Set 2,	Pupil's book, page 410
Problem Set 3,	Pupil's book, page 411
Problem Set 4,	Pupil's book, page 412
Problem Set 5,	Pupil's book, page 413
Problem Set 6,	Pupil's book, page 414
Problem Set 7,	Pupil's book, page 415
Problem Set 8,	Pupil's book, page 416

Solving Problems

Use your own paper. Write the number of the problem.
Write the equations and the sentence that tells the answer.

At the Candy Store

1. Judy bought 6 chocolate bars. Each bar cost 6¢. Find how much Judy's candy bars cost. 36 ¢

2. Sue brought 2 friends with her. She bought 3 bars of baking chocolate for Mother. Each bar cost 18¢. How much did Mother's baking chocolate cost? 54 ¢

3. Bob spent 25¢ for ice cream bars. Each bar cost 5¢. How many ice cream bars did he buy? 5 ice cream bars.

4. Ellen bought 6 lemon drops and 8 orange drops. How many orange and lemon drops did Ellen buy? 14 drops.

5. Jane spent 47¢ for gum and stick candy. She bought 5 packages of gum at 5¢ a package and spent the rest for candy at 2¢ a stick. How many sticks of candy did Jane buy? 11 sticks of candy.

Solving Problems

Use your own paper. Write the number of the problem.

Write the equations and the sentence that tells the answer.

1. Billy had 45¢. He wanted to buy as many 5¢ stamps as he could. How many 5¢ stamps could he buy? 9 stamps.

2. Jeff went in the house to get cookies for his 2 friends and himself. He thought he had 9 cookies but when he got outside and counted them he had 10 cookies. How many cookies should he give to each person? $3 + \frac{1}{3}$ cookies.

3. Kelly had 56 new stamps to paste in his stamp book. Each row in the book has 8 stamps. How many rows of stamps can he paste in the book? 7 rows.

4. Mother bought 4 pencils at 5¢ each. Ann wanted pencils but she didn't like the ones Mother bought so she bought 4 of a different kind. How much did Ann's pencils cost? Not enough information.

5. Father gave Brad 8 golf balls. Two of them had red marks and the rest were marked with green marks. Mother said, "I have 3 golf balls marked with green. You may have them to put with your green ones." How many golf balls marked with green does Brad have? 9 golf balls.

Solving Problems

Use your own paper. Write the number of the problem.

Writ[illegible]e equations and the sentence that tells the answer.

1. John had 50¢. He bought 3 rolls at 7¢ for each roll. How much money did he have then? 29¢

2. Mary bought five 5¢ stamps and six 4¢ stamps. How much did Mary pay for the stamps? 49¢

3. John decided to save nickels. One day he counted his collection and found that he had 9 nickels. How many cents did John have in all? 45¢

4. Susan spent 12¢ for doll dresses and 5¢ for doll shoes. How much change would Susan get if she gave the clerk a quarter to pay for the doll things? 8¢

5. Molly bought a story book for 15¢, a pencil for 5¢, writing paper for 10¢, a doll hat for 10¢, and an eraser for 3¢. How much did Molly spend for writing materials? 18¢

6. David bought 4 kites. Each kite cost exactly the same as the other kites. If David spent 40¢ for the kites, how much did each kite cost? 10¢

Solving Problems

Use your own paper. Write the number of the problem.
Write the equations and the sentence that tells the answer.

1. When Mother cleaned house she washed all of her good dishes. She had 3 sets of dishes and each set had 48 pieces. How many dishes did Mother wash? 144 dishes.

2. Father and Bill helped wash the dishes. It usually took them 30 minutes. Father helped Mother for 20 minutes. How long did Bill work alone with Mother? Not enough information.

3. Mother's cupboards were small so she had to put the 3 sets of dishes in 4 cupboards. If she put the same number of dishes in each cupboard, how many dishes did Mother put in each cupboard? 36 dishes

4. Mother also washed the windows. There were 4 panes of glass in each of the 14 windows Mother washed. How many panes of glass did she wash? 56 panes.

5. It took 10 minutes to wash each window. How long did it take to wash the 14 windows? 140 minutes.

Solving Problems

Use your own paper. Write the number of the problem.

Write the equation and the sentence that tells the answer.

1. Mother made 8 aprons. She gave 4 aprons away. After she made more aprons she had 7 aprons. Find how many more aprons Mother made. 3 more aprons.

2. Jim's team made 3 points in each inning of the game. Find how many points they made in 6 innings. 18 points.

3. Jack had 32 pieces of candy to give to 4 children. Two of the children were girls. If he gave the same number of pieces to each child, how many pieces did each child get? 8 pieces.

4. Sally had 7 stuffed toys. Only 3 of the toys were animals. For her birthday Sally got 5 stuffed toys. Two of them were stuffed dolls and the rest were stuffed animals. Then how many stuffed animals did Sally have? 6 stuffed animals.

5. Three children had birthdays. One cake had 6 candles and another cake had 5 candles. Altogether there were 17 candles. Find how many candles were on the third birthday cake. 6 candles

Solving Problems

Use your own paper. Write the number of the problem.

Write the equations and the sentence that tells the answer.

1. Jean invited 13 girls to her birthday party. She wanted to give each girl 3 balloons. How many balloons did she buy?
   39 balloons. (if Jean didn't have any.)

2. For one of the games Jean wanted to give each of the girls 8 toothpicks. How many toothpicks did Jean need? 104

3. Jean's mother made party favors for all of the girls including Jean. Mother needed 4 sticks of gum and 3 gumdrops for each favor. Find how many sticks of gum she had to buy. 56 sticks of gum.

4. When Jean opened her gifts she found 2 books and 3 boxes of candy. Each box of candy had 18 pieces. How many pieces of candy were in all of the boxes together? 54 pieces of candy.

5. Mother set 3 tables with 5 places at each table. Were there enough places for all of the girls and Mother to be seated? Yes.

6. If Mother used 5 dishes at each place on the tables, how many dishes did she need for all of the places? 75 dishes.

Solving Problems

Use your own paper. Write the number of the problem.

Write the equations and the sentence that tells the answer.

1. Mother bought a large bag of jacks for Beth, Susan and Peggy. On the bag the girls could read, "60 Jacks." Mother gave all of the jacks to the girls. If each girl got the same number of jacks, how many jacks will each girl get? 20 jacks.

2. Mary helped Mother by setting the table for 3 meals every day. How many times did she set the table in a week? 21 times.

3. Jack bought 7 packages of bulbs for his garden. On the outside of 3 packages he read "12 Bulbs." On 4 of the packages he read "10 Bulbs." How many bulbs did Jack buy? 76 bulbs.

4. Mother gave Beth 5 new hair bows and 7 pairs of socks. Grandmother gave Beth six hair bows and another pair of socks. How many hair bows does Beth have now? 11 hair bows.

5. Grandmother had 36 prunes in a bowl. She said to Susan, "Please put all of these into 4 dishes. Be sure to put the same number of prunes in each dish." How many prunes did Susan put in each dish? 9 prunes.

Solving Problems

Use your own paper. Write the number of the problem.

Write the equations and the sentence that tells the answer.

1. Miss Jones brought 3 boxes of chalk and 7 boxes of pencils from the office. Each box held 12 pencils. How many pencils did Miss Jones bring from the office? 84 pencils.

2. Miss Briggs needed chairs for 56 children in the auditorium. Mr. Peterson set up 5 rows of chairs with 12 chairs in each row. How many empty chairs will there be? 4, if only children sat in them.

3. Miss Stone's class was going on a field trip. The children rode in 4 station wagons. Eight children rode in each wagon. Find how many children went on the field trip. 32 children.

4. Mrs. Smith asked John to get enough pencils so that each of the 23 children would have 4 pencils. Find how many pencils John must get. 92 pencils.

5. Miss Kent brought fruit drink for the party. If she could pour 5 glasses full from each bottle, how many glasses could she fill from 7 bottles? 35 glasses.

6. There are 5 classrooms on the first floor of Humbert School and 4 classrooms on the second floor. How many children are in the classrooms on the second floor if each classroom has 31 children in it?
124 Children

VII-10. Factors and clock wheels (Optional)

Objective: To present prime factors and greatest common factors in a concrete situation.

Vocabulary: Greatest common factor.

Background: The supplementary exercises .

Pupil's book, pages 417 - 422: Connect with section 4 of Chapter IV on prime numbers. Review this section with children who do the exercises on clock wheels.

★ Factors and Clock Wheels

1. Write these numbers as products of prime numbers. The first two are done for you.

6 = 2 × 3	4 = 2 X 2
18 = 2 × 3 × 3	8 = 2 X 2 X 2
10 = 2 X 5	7 = 7
14 = 2 X 7	30 = 5 X 2 X 3
9 = 3 X 3	100 = 2 X 2 X 5 X 5

2. We say that 10 and 14 have the common factor 2 since 2 is a factor of both 10 and 14. What common factor do 6 and 9 have? 3 (Since 1 is always a common factor, there is no need to mention it.) Look up above at the factors of 18 and 30. What common factors do 18 and 30 have? 2, 3, 6 The greatest common factor of 18 and 30 is 2 × 3 or 6.

3. Find the greatest common factor of each pair of numbers:

14 = 2 × 7
21 = 3 × 7
Greatest common factor: 7

35 = 5 × 7
20 = 2 × 2 × 5
Greatest common factor: 5

12 = 2 × 2 × 3
9 = 3 × 3
Greatest common factor: 3

10 = 2 X 5
15 = 3 X 5
Greatest common factor: 5

9 = 3 X 3
8 = 2 X 2 X 2
Greatest common factor: 1

4. Find the greatest common factor of each pair of numbers:

7 = 7
21 = 3 X 7
Greatest common factor: 7

20 = 2 X 2 X 5
9 = 3 X 3
Greatest common factor: 1

20 = 2 × 2 × 5
28 = 2 × 2 × 7
Greatest common factor: 2 × 2 = 4

12 = 2 X 2 X 3
18 = 2 X 3 X 3
Greatest common factor: 2 X 3 = 6

30 = 2 X 3 X 5
40 = 2 X 2 X 2 X 5
Greatest common factor: 2 X 5 = 10

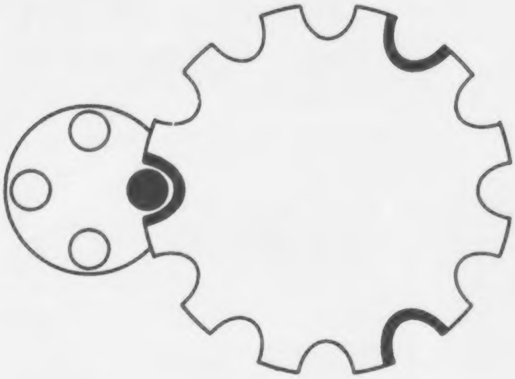
16 = 2 X 2 X 2 X 2
26 = 2 X 13
Greatest common factor: 2

44 = 2 X 2 X 11
20 = 2 X 2 X 5
Greatest common factor: 2 X 2 = 4

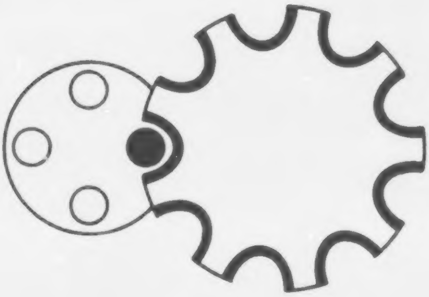
12 = 2 X 2 X 3
25 = 5 X 5
Greatest common factor: 1

33 = 3 X 11
6 = 2 X 3
Greatest common factor: 3

5. Here is a pair of wheels from an old clock. Imagine them turning. Outline with a black crayon all the notches on wheel B which are touched by the black peg.

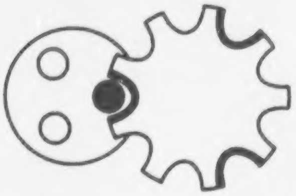


In the picture below, the big wheel has 9 notches instead of 12. Again, outline in black the notches touched by the black peg.

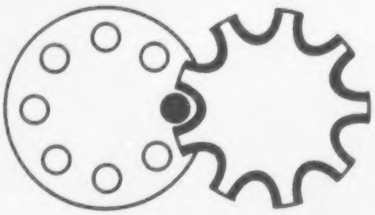


Can you explain why the black peg touched only 3 notches in the first case, but touched all 9 in the second case? The examples on the next page may help.

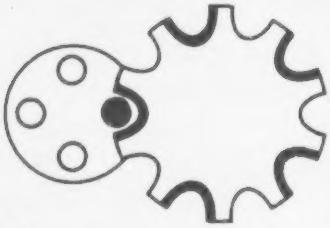
6. Fill in the number of notches and the number of pegs. Then outline in black the notches touched by the black peg as the wheels turn.



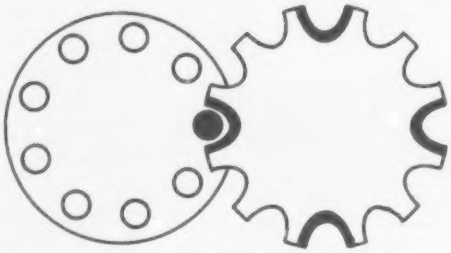
3 pegs, 9 notches



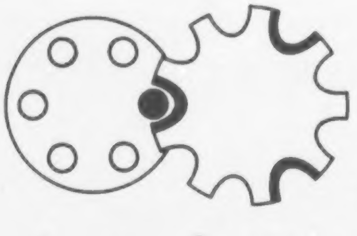
8 pegs, 9 notches



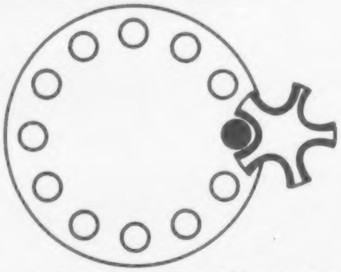
4 pegs, 10 notches



9 pegs, 12 notches



6 pegs, 9 notches



12 pegs, 5 notches

Now look at what you have done. Can you think of a rule to tell you when the black peg will touch all the notches and when it won't? (Think about common factors.)

7. Now try some without pictures. Mark with X those examples in which the black peg would touch all the notches.

3 pegs, 5 notches X

6 pegs, 12 notches ____

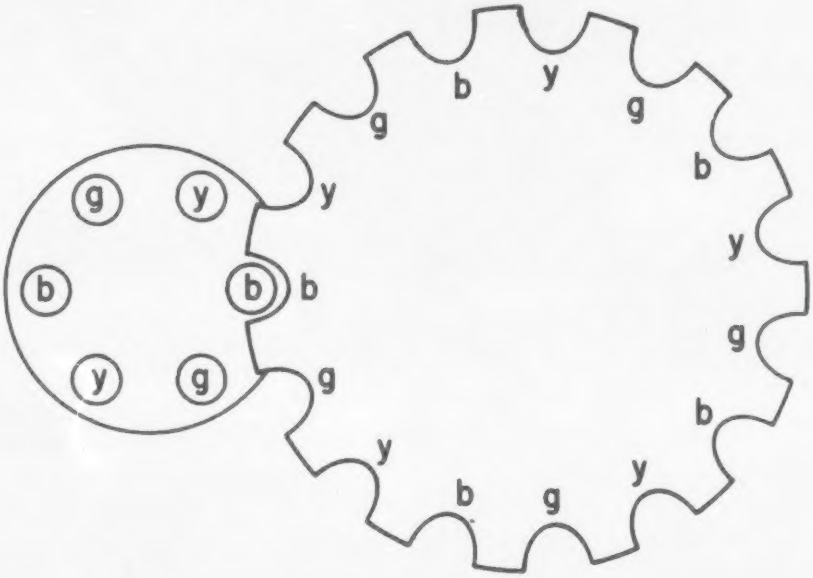
7 pegs, 11 notches X

6 pegs, 14 notches ____

9 pegs, 24 notches ____

15 pegs, 16 notches X

8. In this picture color the pegs and notches in such a way that no peg of one color ever touches a notch of another color. Use as many different colors as you can.



Number of pegs: 6

Number of notches: 15

Greatest common factor of these numbers: 3

Number of different colors: 3

Now color all the wheels on the pages you did before in the same way that you colored these.

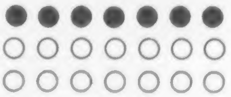
9. Suppose wheel A has 28 pegs of which one is black, and wheel B has 100 notches. How many notches does the black peg touch as the wheels turn? ___25___ What is the greatest number of different colors you could use to color the pegs and notches so that no peg ever touches a notch of a different color? ___4___ How many pegs would you have of each color? ___7___ How many notches would you have of each color? ___25___ Don't try to draw a picture; just work with the numbers.

Chapter VIII

RATIONAL NUMBERS

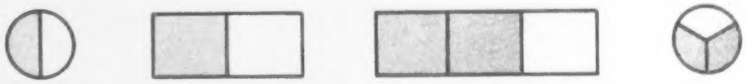
Background

Meaning for the rational numbers is first developed by partitioning a set of objects into equivalent subsets and associating a number with one or more subsets with respect to the set, regarded as a unit. For example:

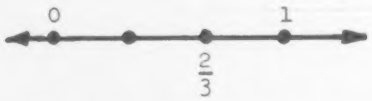


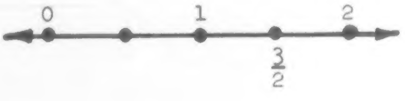
the shaded objects represent 1/3 of the set.

Also, children learn to associate numbers like 1/2 and 2/3 with a shaded portion of a figure, where the figure is identified as the unit region, as illustrated in the following figures:



These ideas used with regions are extended to the number line and unit segments. For example, in order to locate the point corresponding to 2/3 we mark off the unit segment into 3 congruent parts. We then count off 2 of them. If we have separated each unit interval into 2 congruent parts and count off 3 of them, we have located the point which we would associate with 3/2.





Names for Rational Numbers.

The symbol "$\frac{m}{n}$" is called a *fraction*. A fraction is a symbol which results from writing a numeral, drawing a bar under it, and writing a numeral beneath the bar. In a fraction the number named above the bar is called the *numerator*, while the number named below the bar is called the *denominator*. When the numerator of a fraction is a whole number and the denominator is a counting number, then the number which this fraction designates, or names, is called rational--because it has to do with *ratios*, not reason. All whole numbers are rational numbers. The number 5, for instance is rational because it can be designated by the fraction, $\frac{5}{1}$ (or $\frac{10}{2}$, or $\frac{15}{3}$, etc.) with whole number numerator and denominator.

VIII-1. Rational numbers associated with parts of regions

Objective: To review the idea of associating rational numbers with parts of regions.

Vocabulary: (No new words.)

Materials: Unit circular regions of different sizes, rectangular regions of different sizes, triangular regions (isosceles and equilateral) of different sizes. (These may be of felt and used on flannelboard, or construction paper displayed on the bulletin board, or drawn on the chalkboard.)

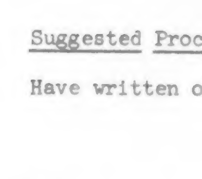
Suggested Procedure:

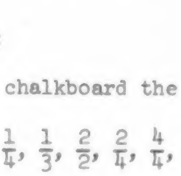
Have written on the chalkboard the numerals--

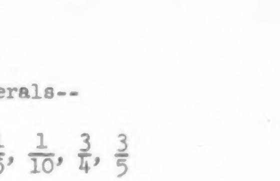
$$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{2}{2}, \frac{2}{4}, \frac{4}{4}, \frac{3}{3}, \frac{1}{5}, \frac{1}{10}, \frac{3}{4}, \frac{3}{5}$$

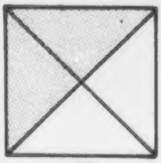
Show a unit region and then indicate a part of this same region. Ask children what number is associated with the part of the region if 1 is associated with the region. Ask someone to find the name of the number on the chalkboard. (e.g., one half of a circular region: $\frac{1}{2}$.)

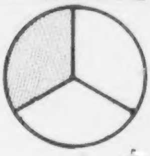
Continue with other parts of units, first exhibiting the unit and then the part of the unit. Have children tell what number is associated with each of the following parts of regions.

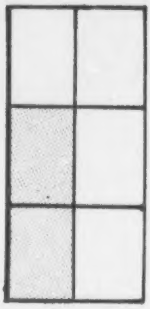


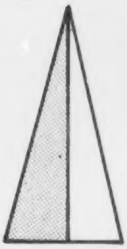


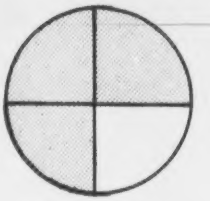












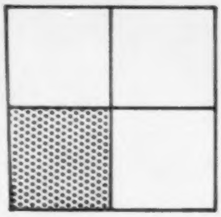
As they observe each figure, have them first identify the unit, and then the part of the unit that is shown or shaded.

Pupil's book, pages 423 - 424: Have children ring the fraction which tells what part of the region is shaded.

Pupil's book, page 425: Discuss the chart with the children. In the first box, they are to put the number of parts shaded; in the second box, the number of congruent parts into which the region has been divided; in the third box, the fraction which names the shaded area.

Rational Numbers

Ring the fraction that tells what part is shaded.

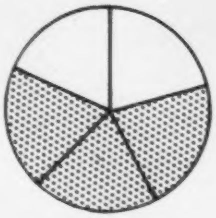


$\frac{3}{4}$

$\frac{2}{4}$

[✓ $\frac{1}{4}$]

$\frac{4}{1}$

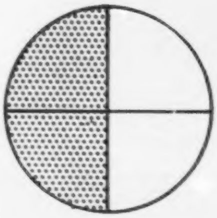


[✓ $\frac{3}{5}$]

$\frac{5}{3}$

$\frac{2}{5}$

$\frac{5}{5}$

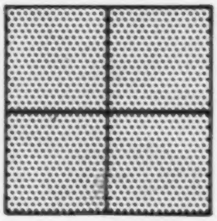


$\frac{3}{5}$

[✓ $\frac{2}{4}$]

$\frac{4}{2}$

$\frac{1}{3}$

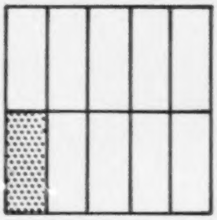


$\frac{1}{4}$

[✓ $\frac{4}{4}$]

$\frac{3}{4}$

$\frac{2}{4}$

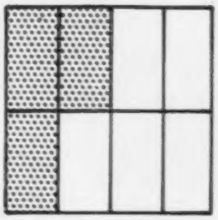


[✓ $\frac{1}{10}$]

$\frac{10}{1}$

$\frac{1}{5}$

$\frac{1}{3}$



$\frac{5}{8}$

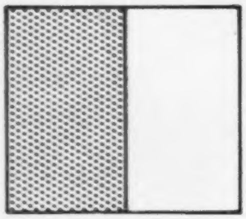
[✓ $\frac{3}{8}$]

$\frac{1}{4}$

$\frac{3}{4}$

Rational Numbers

Ring the fraction that tells what part is shaded.

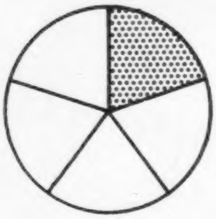


$\frac{1}{3}$

[✓ $\frac{1}{2}$]

$\frac{2}{1}$

$\frac{2}{2}$

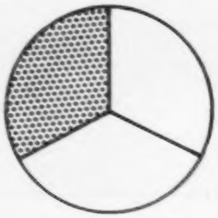


$\frac{5}{1}$

[✓ $\frac{1}{5}$]

$\frac{4}{5}$

$\frac{2}{5}$

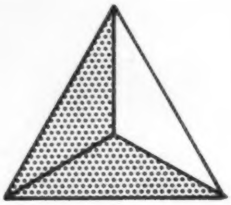


$\frac{2}{3}$

[✓ $\frac{1}{3}$]

$\frac{3}{1}$

$\frac{1}{2}$

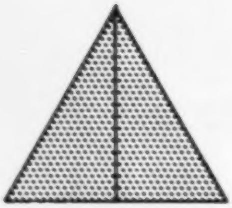


$\frac{1}{3}$

[✓ $\frac{2}{3}$]

$\frac{3}{2}$

$\frac{3}{3}$

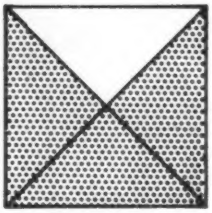


$\frac{1}{2}$

$\frac{1}{4}$

[✓ $\frac{2}{2}$]

$\frac{2}{4}$



$\frac{1}{4}$

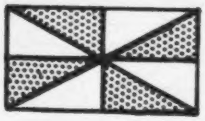
$\frac{4}{3}$

[✓ $\frac{3}{4}$]

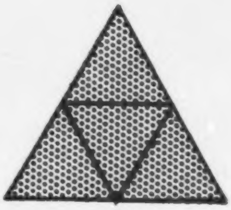
$\frac{2}{3}$

Rational Numbers

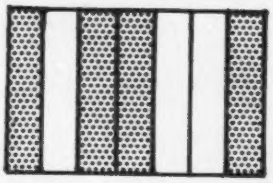
Complete the chart.



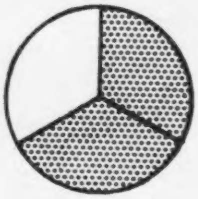
A



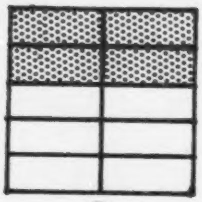
B



C



D



E

Figure	Parts Shaded	Congruent Parts of Unit	Name of Number of the Shaded Area
A	4	8	$\frac{4}{8}$
B	4	4	$\frac{4}{4}$
C	4	7	$\frac{4}{7}$
D	2	3	$\frac{2}{3}$
E	4	10	$\frac{4}{10}$

VIII-2. Rational numbers associated with subsets of a set

Objective: To review the idea of associated rational numbers with a part of a set of objects.

Vocabulary: (No new words.)

Materials: Materials to be used on the flannel board, small objects to be used by children.

Suggested Procedure:

In the preceding books, children have learned to recognize that $\frac{1}{2}$ of a set of 6 is 3, $\frac{1}{4}$ of a set of 4 is 1, etc. Display on the flannel board or magnetic board a set of 8 disks. (Children may pretend that these objects represent balls, cookies, or some other object.) Then ask someone to arrange the set in rows of 4.

> Now, let's imagine that Joe takes one row.
>
> What number suggests the part of the set? ($\frac{1}{2}$.)
>
> What number suggests the number of cookies he took? (4.)
>
> What mathematical sentence can be written?
>
> $$\frac{1}{2} \text{ of } 8 = 4$$

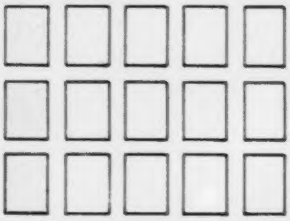
Continue with other sets of objects using $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.

Pupil's book, pages 426 and 427 provide practic for reviewing these ideas.

Rational Numbers and Sets of Objects

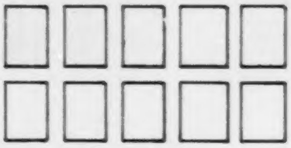
Name the rational number suggested by shaded objects in each set.

1.



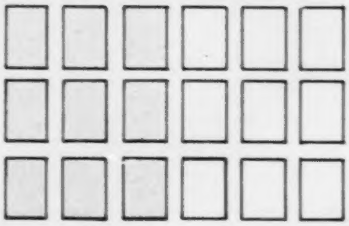
$\frac{1}{5}$

2.



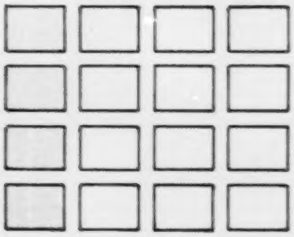
$\frac{1}{2}$

3.



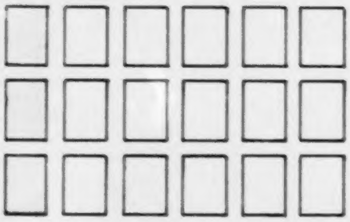
$\frac{1}{2}$

4.



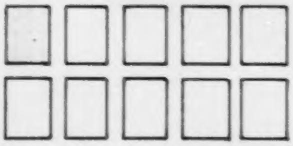
$\frac{1}{4}$

5.



$\frac{1}{6}$

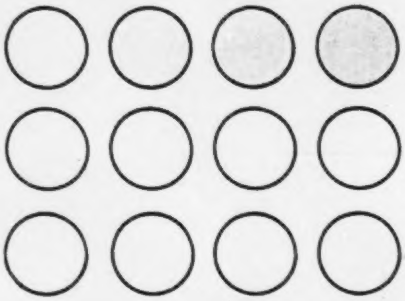
6.



$\frac{1}{10}$

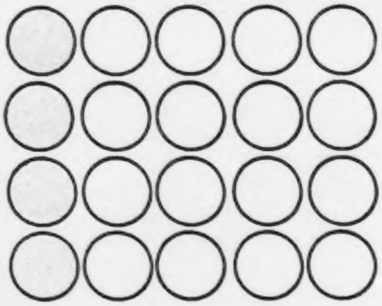
Rational Numbers and Sets of Objects

Complete the sentences.



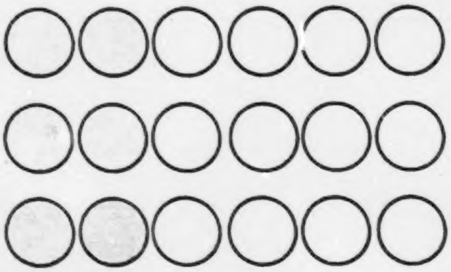
$\frac{1}{3}$ of 12 = 4

12 = 3 × 4



$\frac{1}{5}$ of 20 = 4

20 = 4 × 5



$\frac{1}{3}$ of 18 = 6

18 = 3 × 6

VIII-3. Identifying a subset as a part of a set

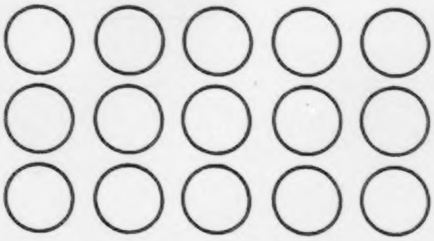
Objective: To associate a rational number with a subset of a set.

Vocabulary: (No new terms.)

Materials: Materials for flannel board; manipulative materials.

Suggested Procedure:

Place a set of objects on the flannelboard, such as



Ask how many objects are in the set. Then say:

> Suppose we think of these disks as balls.
> Suppose Tom takes 5 of these balls
> and Joe takes 5 of these balls.
> Each boy gets what part of the set of balls? ($\frac{1}{3}$.)
> Together, they get what part of the set? ($\frac{2}{3}$.)
> What part of the set is left? ($\frac{1}{3}$.)
> What sentence can we write?
>
> $\frac{2}{3}$ of 15 is 10.
>
> $\frac{1}{3}$ of 15 is 5.

Help children to see that $\frac{1}{3}$ of 15, or 5, is one of 3 equivalent subsets of 15. If we think of 2 of these subsets, we call that part $\frac{2}{3}$ of 15, or 10.

Show other examples using rational numbers:

$\frac{2}{2}$, $\frac{3}{4}$, $\frac{3}{3}$, $\frac{3}{5}$, $\frac{2}{6}$, $\frac{5}{6}$, etc.

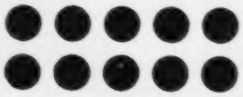
To find $\frac{5}{6}$ of 18, for instance, children will first think of partitioning 18 into 6 equivalent subsets. One of these subsets has 3 members. ($\frac{1}{6}$ of 18 is 3.) Five such subsets would have 5 × 3 or 15 members. ($\frac{5}{6}$ of 18 = 15.)

Give much additional practice with flannel board materials, manipulative materials, or chalkboard drawings.

Pupil's book, pages 428 - 430: These pages may be used for class discussion or independent work.

Rational Numbers

Complete:



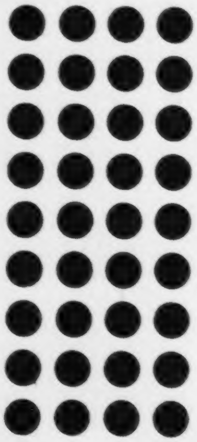
$10 = \underline{2} \times 5$

$\frac{1}{2}$ of $10 = \underline{5}$

$\frac{1}{5}$ of $10 = \underline{2}$

$\frac{3}{5}$ of $10 = \underline{6}$

$\frac{5}{5}$ of $10 = \underline{10}$



$36 = \underline{9} \times 4$

$\frac{1}{9}$ of $36 = \underline{4}$

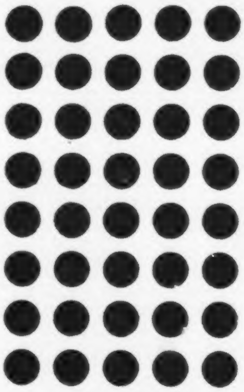
$\frac{3}{9}$ of $36 = \underline{12}$

$\frac{5}{9}$ of $36 = \underline{20}$

$\frac{7}{9}$ of $36 = \underline{28}$

Rational Numbers

Complete:



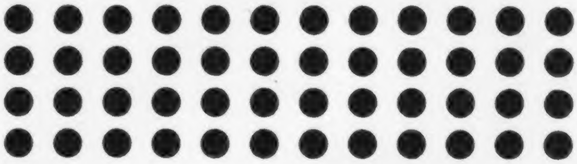
40 = 8 × 5

$\frac{1}{8}$ of 40 = 5

$\frac{3}{8}$ of 40 = 15

$\frac{6}{8}$ of 40 = 30

$\frac{8}{8}$ of 40 = 40



48 = 4 × 12

$\frac{1}{4}$ of 48 = 12

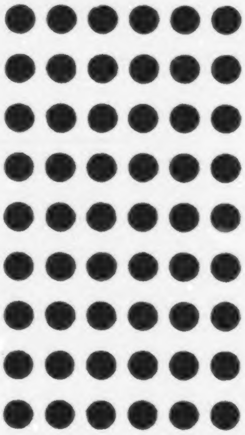
$\frac{1}{12}$ of 48 = 4

$\frac{1}{2}$ of 48 = 24

$\frac{12}{12}$ of 48 = 48

Rational Numbers

Complete:



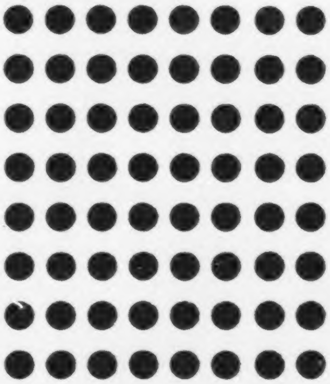
54 = 9 × 6

$\frac{1}{9}$ of 54 is 6

$\frac{1}{6}$ of 54 is 9

$\frac{6}{6}$ of 54 is 54

$\frac{9}{9}$ of 54 is 54



64 = 8 × 8

$\frac{1}{8}$ of 64 is 8

$\frac{3}{8}$ of 64 is 24

$\frac{4}{8}$ of 64 is 32

$\frac{5}{8}$ of 64 is 40

$\frac{8}{8}$ of 64 is 64

VIII-4. Finding a part of a set

Objective: To use rational numbers to express the relationship between two numbers.

Vocabulary: (No new words.)

Materials: (Same as in the preceding lesson.)

Suggested Procedure:

Suggest a problem situation such as the following:

> Suppose Ann has 10 books to read.
> She reads 2 of them.
> What part of the 10 books has she read?

Place cards on the flannelboard to represent the ten books. Then have children designate those that she has read. Ask them to find what part of the set she has read. Let children suggest ways of identifying the number that tells what part she read. ($\frac{2}{10}$, $\frac{1}{5}$.)

Then write the mathematical sentence for this problem

2 is ____ of 10. ($\frac{1}{5}$ or $\frac{2}{10}$.)

Continue with other problem situations where the number of a subset is known and the number of the set is known. Children are then to associate a rational number with the part of the set represented by the subset.

Then, continue with other problem situations such as illustrated by the following:

> You have 10 pennies.
> You spend 6 of them.
> What part of your money did you spend?

Spent

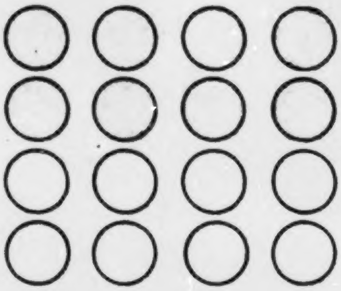
●●●●●●○○○○

Have children suggest ways of partitioning the set. Some may suggest making 2 sets, but find that this does not help answer the question. Ask for other ways that could be used. Hopefully, children will suggest 10 subsets, whereby the number would be named by 6/10. Others may suggest five sets with two pennies in each set, whereby they name the number by the fraction 3/5. Use other situations each time selecting a subset which cannot be related to the set by using a unit fraction, that is, 1/2, 1/3, 1/4, etc.

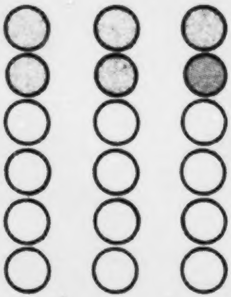
Pupil's book, pages 431 to 433: Children are to name the rational number suggested by the relation between a subset of a set and that set. Do not require that they use $\frac{1}{2}$ rather than $\frac{8}{16}$, $\frac{2}{3}$ instead of $\frac{4}{6}$, etc.

Rational Numbers and Sets of Objects

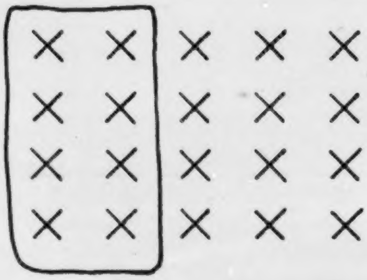
Complete:



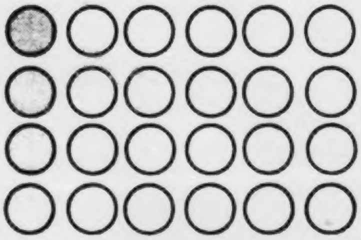
8 is $\frac{8}{16}$ of 16. or $\frac{1}{2}$ of 16



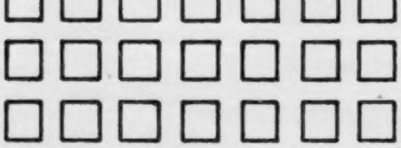
6 is $\frac{6}{18}$ of 18. or $\frac{1}{3}$ of 18



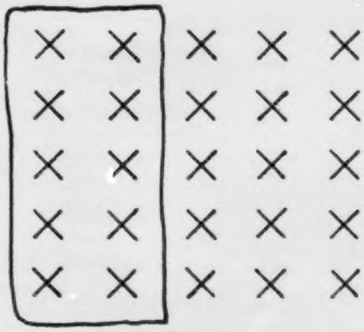
8 is $\frac{8}{20}$ of 20. or $\frac{2}{5}$ of 20



4 is $\frac{4}{24}$ of 24. or $\frac{1}{6}$ of 24



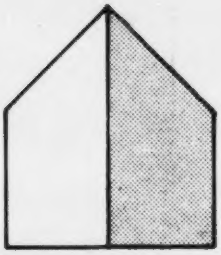
6 is $\frac{6}{21}$ of 21. or $\frac{2}{7}$ of 21



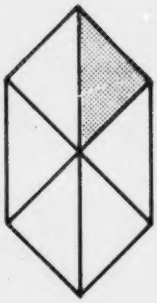
10 is $\frac{10}{25}$ of 25. or $\frac{2}{5}$ of 25

Rational Numbers -- Review

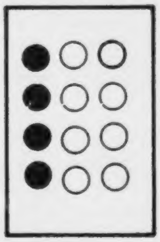
For each figure, write a fraction which names the rational number suggested by the shaded part of the picture.



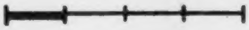
$\frac{1}{2}$



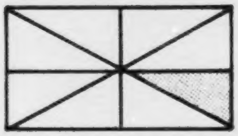
$\frac{1}{6}$



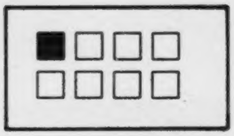
$\frac{1}{3}$



$\frac{1}{4}$



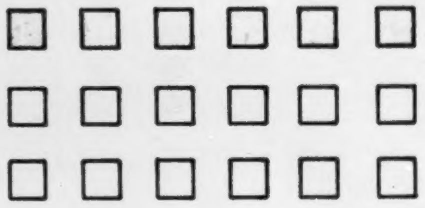
$\frac{1}{8}$



$\frac{1}{8}$

Rational Numbers and Sets of Objects

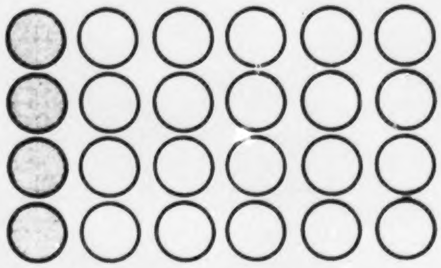
Complete:



6 is $\frac{1}{3}$ of 18.

3 is $\frac{1}{6}$ of 18.

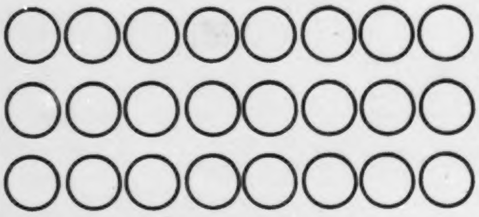
9 is $\frac{1}{2}$ of 18.



4 is $\frac{1}{6}$ of 24.

6 is $\frac{1}{4}$ of 24.

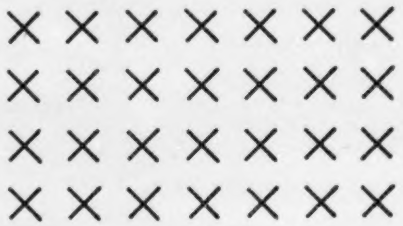
12 is $\frac{1}{2}$ of 24.



3 is $\frac{1}{8}$ of 24.

8 is $\frac{1}{3}$ of 24.

21 is $\frac{7}{8}$ of 24.



7 is $\frac{1}{4}$ of 28.

4 is $\frac{1}{7}$ of 28.

14 is $\frac{1}{2}$ of 28.

28 is $\frac{28}{28}$ of 28.

VIII-5. Rational numbers describe points of the number line

Objective: To review and extend the idea of rational numbers describing points on the number line.

Vocabulary: (No new words.)

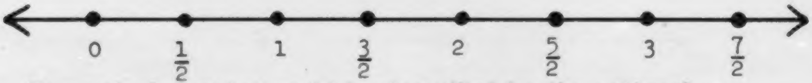
Materials: Line on chalkboard with 5 equally spaced points (1 foot apart).

Suggested Procedure:

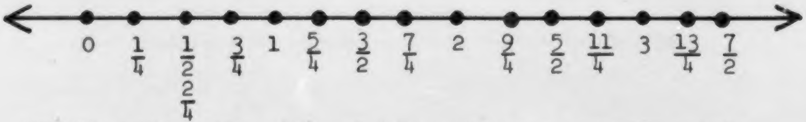
Label the left point on the line the 0-point. Then have a child label the other marked points by writing 1, 2, 3, below the line. Now, let us use the rational numbers to label points on the line. What point can be labeled by 1/2 ? Guide children by suggesting the separation of the first unit segment into two congruent parts.

Now let us find other points where the segments are congruent to the one whose end points are marked by 0 and 1/2. Mark these points on the number line.

Then have children suggest rational numbers that will describe these points. The line may appear as follows:



Then ask what point could be described by the rational number 1/4. Again, observe that there is a smaller segment. Again, lay off segments equivalent to this point. Identify the rational numbers that describe these points, such as,

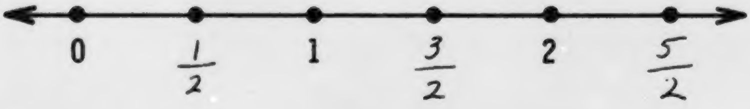


Pupil's Book, pages 434 and 435: Children describe points of number line using both the whole numbers and the rational numbers.

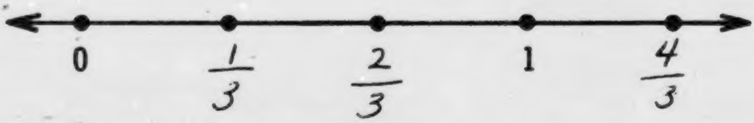
Rational Numbers Describe Points On Number Line.

Label each point marked, using a rational number.

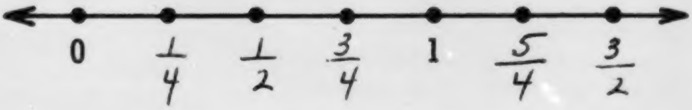
A.



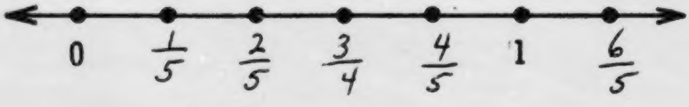
B.



C.


0
1/4
1/2
3/4
1
5/4
3/2

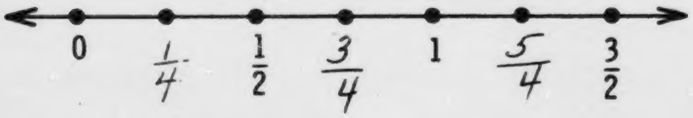
D.


0
1/5
2/5
3/4
4/5
1
6/5

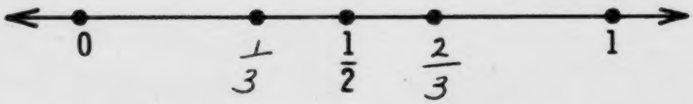
Rational Numbers Describe Points On Number Line

Label each point marked, using a rational number.

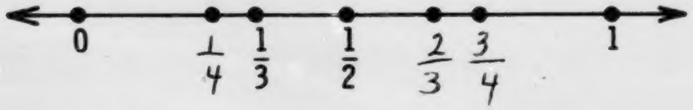
A.


0
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{3}{4}$
1
$\frac{5}{4}$
$\frac{3}{2}$

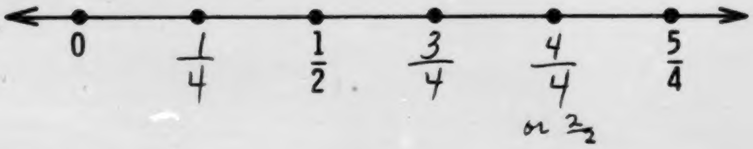
B.


0
$\frac{1}{3}$
$\frac{1}{2}$
$\frac{2}{3}$
1

C.


0
$\frac{1}{4}$
$\frac{1}{3}$
$\frac{1}{2}$
$\frac{2}{3}$
$\frac{3}{4}$
1

D.


0
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{3}{4}$
$\frac{4}{4}$
or $\frac{2}{2}$
$\frac{5}{4}$

VIII-6 Order among rational numbers

Objective: To compare numbers named by fractions having the same numerators.

Vocabulary: (No new words.)

Materials: Circular and rectangular regions. Line on the chalkboard.

Suggested Procedure:

Display on the flannelboard a circular region. Below this region show 1/2 unit region, 1/3 unit region, 1/4 unit region, 1/5 unit region and 1/6 unit region. Ask what number can be associated with each region. Then ask which is greater 1/2 or 1/3, etc. Using pairs of numbers record the responses

$$\frac{1}{2} > \frac{1}{3}$$

$$\frac{1}{3} > \frac{1}{6}$$

$$\frac{1}{4} > \frac{1}{6}$$

$$\frac{1}{3} > \frac{1}{4}$$

etc.

Also ask which is less 1/2 or 1/3, etc. Record the responses

$$\frac{1}{3} < \frac{1}{2}$$

$$\frac{1}{4} < \frac{1}{3}$$

etc.

Use rectangular regions in the same way. Then ask if someone can suggest how these could be written starting with the greatest number and going to the least. Then reverse the order beginning with the least and going to the greatest.

Then refer to the line on the chalkboard. Label the points using whole numbers.

Then ask where they would expect 1/2 to be? $\frac{1}{3}$, $\frac{1}{4}$

Return to the circular region again. Exhibit below the unit, 1/4 unit region, 2/3 unit region, 3/4 unit region, 4/4 unit region, and 5/4 unit region. Again identify the number that can be associated with each part of the region. Then write sentences about these numbers using the words, greater than and less than.

Using another line on the chalkboard, label the points using whole numbers then separate each unit into 4 equivalent parts. Ask what number can be associated with each of these points. Record the name of the number below the point.

Pupil's book, pages 436 - 437: These pages use shaded regions to help children compare rational numbers.

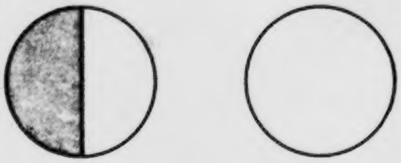
Pupil's book, pages 438 and 439: Provide a follow-up of the class experience with opportunity for each child to work on his own. If some children need help, you may wish to do some of the exercises together with those children.

Order Among Rational Numbers

Name the rational number suggested by each shaded region.

Then complete the sentence.

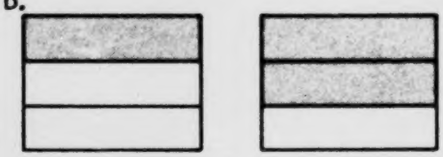
A.



$\frac{1}{2}$ $\frac{0}{2}$

$\frac{0}{2} < \frac{1}{2}$

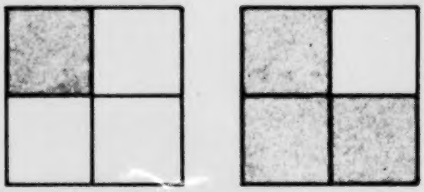
B.



$\frac{1}{3}$ $\frac{2}{3}$

$\frac{2}{3} > \frac{1}{3}$

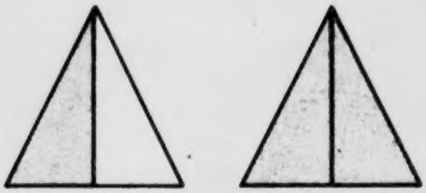
C.



$\frac{1}{4}$ $\frac{3}{4}$

$\frac{3}{4} > \frac{1}{4}$

D.



$\frac{1}{2}$ $\frac{2}{2}$

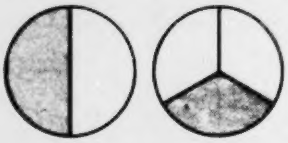
$\frac{1}{2} < \frac{2}{2}$

Order Among Rational Numbers

Name the rational number suggested by each shaded region.

Then complete the sentence.

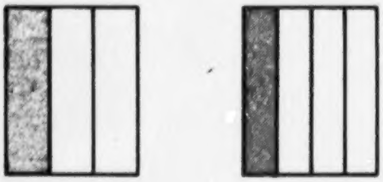
E.



$\frac{1}{2}$ $\frac{1}{3}$

$\frac{1}{3} < \frac{1}{2}$

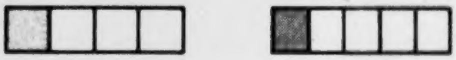
F.



$\frac{1}{3}$ $\frac{1}{4}$

$\frac{1}{3} > \frac{1}{4}$

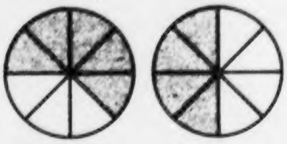
G.



$\frac{1}{4}$ $\frac{1}{5}$

$\frac{1}{4} > \frac{1}{5}$

H.



$\frac{5}{8}$ $\frac{4}{8}$

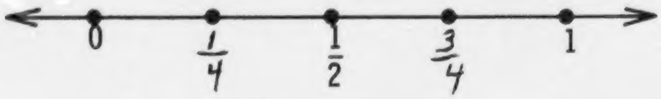
$\frac{4}{8} < \frac{5}{8}$

Order Among Rational Numbers

Label each point using a rational number name.

Then complete each sentence using >, <, or =.

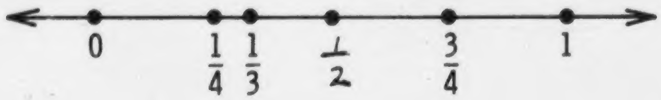
A


0
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{3}{4}$
1

$\frac{1}{4}$ < $\frac{1}{2}$

$\frac{1}{4}$ > 0

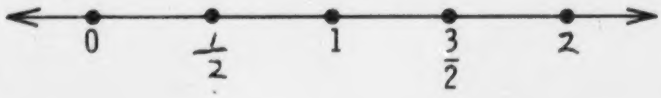
B.


0
$\frac{1}{4}$
$\frac{1}{3}$
$\frac{1}{2}$
$\frac{3}{4}$
1

$\frac{3}{4}$ > $\frac{1}{3}$

$\frac{2}{4}$ = $\frac{1}{2}$

C.


0
$\frac{1}{2}$
1
$\frac{3}{2}$
2

$\frac{3}{2}$ > 1

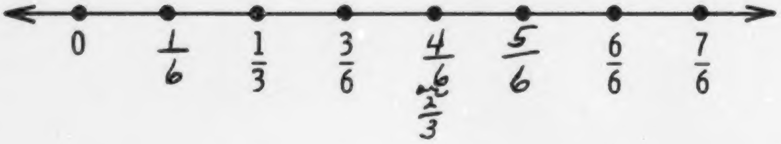
$\frac{1}{2}$ < $\frac{3}{2}$

Order Among Rational Numbers

Label each point using a rational number name.

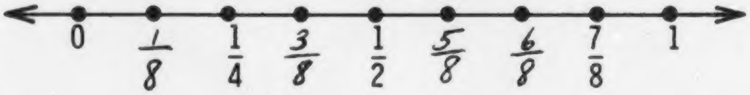
Then complete each sentence using >, < or =.

D.


0
$\frac{1}{6}$
$\frac{1}{3}$
$\frac{3}{6}$
$\frac{4}{6}$
$\frac{2}{3}$
$\frac{5}{6}$
$\frac{6}{6}$
$\frac{7}{6}$

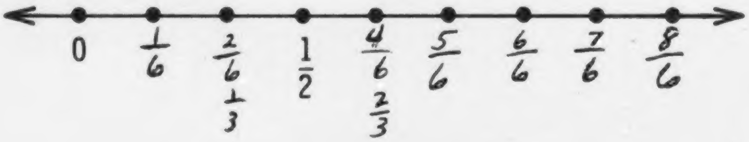
$\frac{3}{6}$ < $\frac{6}{6}$ $\frac{4}{6}$ > $\frac{2}{6}$

E.


0
$\frac{1}{8}$
$\frac{1}{4}$
$\frac{3}{8}$
$\frac{1}{2}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{7}{8}$
1

$\frac{7}{8}$ < $\frac{8}{8}$ $\frac{1}{2}$ = $\frac{4}{8}$ $\frac{2}{4}$ = $\frac{1}{2}$

F.


0
$\frac{1}{6}$
$\frac{2}{6}$
$\frac{1}{3}$
$\frac{1}{2}$
$\frac{4}{6}$
$\frac{2}{3}$
$\frac{5}{6}$
$\frac{6}{6}$
$\frac{7}{6}$
$\frac{8}{6}$

$\frac{1}{2}$ = $\frac{3}{6}$ $\frac{1}{3}$ = $\frac{2}{6}$ $\frac{1}{2}$ > $\frac{1}{3}$

Chapter IX

DIVISION

Background

In the background for Chapter VII, we noted the beginning concepts of division and its relation to multiplication. You may wish to reread that section at this time. If we restricted the set of numbers to the whole numbers, we observed that division was not always possible. Now that children have some understanding of the rational numbers, we turn again to the idea of division and complete division.

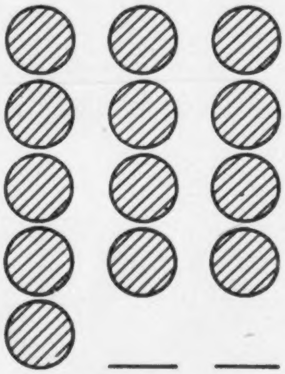
Let us consider the numbers 13 and 3 and division, where 13 is to be divided by 3. We can express the quotient of these numbers as

$$13 \div 3, \text{ or } \frac{13}{3}$$

$$13 \div 3 = \frac{13}{3}$$

At this stage of understanding we associate 13 with a set of objects. 3 can be associated either with (a) the number in each row or (b) the number of rows. Let us consider both.

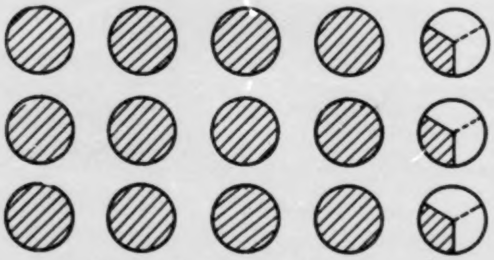
(a) First we arrange 13 into rows with 3 objects in each row



We observe that there are 4 rows of 3 and $\frac{1}{3}$ of another row.

We write: $13 \div 3 = 4 + \frac{1}{3}$ or $\frac{13}{3} = 4 + \frac{1}{3}$

(b) Now let us rearrange the 13 objects into 3 rows.



We observe that there are 3 rows with $(4 + \frac{1}{3})$ objects in each row.

We write: $13 \div 3 = 4 + \frac{1}{3}$ or $\frac{13}{3} = 4 + \frac{1}{3}$.

The purpose of this chapter is to provide materials for helping children complete the division, using rational numbers. Many children can be expected to do their thinking with the help of sets of objects. They learn to associate the remainder with a subset of a set whose number is the divisor. They identify the subset as a part of the set. Suppose that the divisor is 3 and the remainder is 1, then 1 is $\frac{1}{3}$ of 3. Or, if the remainder is 4 and the divisor is 8, then 4 is $\frac{4}{8}$ or $\frac{1}{2}$ of 8, etc. Although it is hoped that background is provided for this kind of thinking in the preceding chapter, opportunity is provided for continued work with this idea in this chapter.

Techniques of Division. Children learn to use a computation form for finding the quotient. If the numbers are 22 and 4 where 22 is to be divided by 4, then

$$\begin{array}{r|l} 4\overline{)\,22} & \\ 20 & 5 \\ \hline 2 & \end{array}$$

Using whole numbers the division cannot be completed. Children may say that twenty-two is equal to five times four plus two or write $22 = (5 \times 4) + 2$.

Using rational numbers, the division is completed and the quotient is $(5 + \frac{2}{4})$

$$\frac{22}{4} = 5 + \frac{2}{4}$$

Teaching note. Although this is a departure from common practice where the division is restricted to whole numbers or where division is restricted to numbers where the first is a multiple of the second, we believe that this departure will help children to come to a better understanding of the idea of division.

IX-1. Division concepts

Objective: To review basic division concepts.

Vocabulary: (Review) Factor, product, division, quotient, divisor.

Materials: Manipulative objects for children, materials for flannel board.

Suggested Procedure:

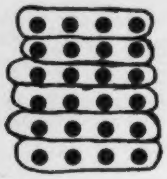
Arrange 24 flannel butterflies at random on the flannel board. Propose a problem situation.

> Jimmy is going to bring his butterfly collection to school. He has 24 butterflies and he plans to put 4 butterflies in a box. How many boxes does he need?
>
> It will be easy to answer this question if we arrange the butterflies in an array. Do we know how many butterflies are to be put in one box? (Yes, 4.)
>
> Then do we know how many elements will be in one row of our array? (Yes, 4.)

Ask some child to arrange the butterflies in an array.

> How many rows did we need? (6.)
>
> What equation can we write about this array? ($6 \times 4 = 24$.)
>
> But when we began we didn't know the number of rows. What equation would we have written then? ($n \times 4 = 24$.)
>
> Now do we know how many boxes Jimmy needed? (Yes, 6.)

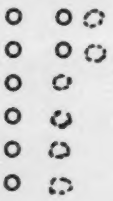
Represent the array on the chalkboard. Ring each subset of 4 members.



Ask the children if the array shows that there are 6 fours in 24. (Yes.)

> In this problem we found "n" by partitioning a set of 24 into equivalent subsets of 4 elements. The array showed that there are 6 fours in 24 or that 24 ÷ 4 = 6.
>
> That was an easy problem because we knew the number of members in each row. There are some problems we have to solve and we don't know the number of elements in a row. Suppose I said that Jimmy had 24 butterflies and 6 boxes. How can we find the number of butterflies he would put in a box?

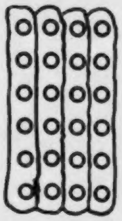
A natural way is to set up an array with 6 rows and then observe the number of elements in each row.



If none of the children suggest this method, describe the situation as you arrange the objects on the flannel board.

> What equation describes this array?
> ($6 \times 4 = 24$.)
> But when we began we didn't know the number of elements in each row.
> What equation would we have written then?
> ($6 \times n = 24$.)

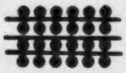
Represent the array on the chalkboard. Ring each subset of 6 members.

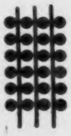


Ask the children if the array shows that there are 4 sixes in 24. (Yes.)

> In this problem we found "n" by partitioning a set of 24 into 6 equivalent subsets of 4 elements. The array also showed that there are 4 sixes in 24. We may think of the equation $24 \div 4 = 6$ as stating that the number of fours in 24 is 6, or that 4 is the number of sixes in 24.

Direct the children's attention to the arrays on the chalkboard. Make sure that they understand they can partition a set in two ways to represent a division equation, such as: $24 \div 4 = 6$.

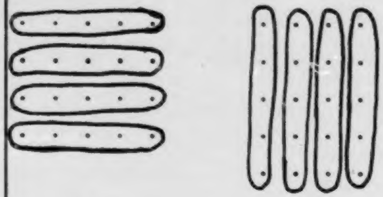




Pupil's book, page 440: This page gives practice in partitioning an array in two ways to represent a division equation. Children are to draw two arrays to represent each equation.

Using Arrays to Show Division

The first exercise has been done for you.

 $20 \div 4 = 5$	$12 \div 3 =$ 4
$15 \div 3 =$ 5	$\frac{21}{3} =$ 7
$\frac{32}{8} =$ 4	$16 \div 8 =$ 2
$18 \div 3 =$ 6	$30 \div 5 =$ 6

Dividing

Begin with a problem situation.

> Mrs. Brown gave 12 cookies to her three children.
> How many cookies did each child receive?

Ask the children what multiplication equation describes this situation. ($3 \times n = 12$.) Remind them that they can find what number "n" is by partitioning the set of 12 into equivalent subsets of 3. Draw a 3 by 4 array on the chalkboard and have a child show the two ways in which the partitioning can be done.

Suggest that it is possible to solve the equation $3 \times n = 12$ in another way. If no one suggests division, point out that in this equation we are multiplying some number by 3 to get 12 as the product. Ask how it is possible to undo multiplying by 3. (Dividing by 3.) So, $3 \times n = 12$ can be written $12 \div 3 = n$. Remind the children that we speak of partitioning a set into equivalent subsets but that we say we divide a number by another number.

Ask if anyone remembers another way of writing $12 \div 3$. If not, continue:

> We can write "Twelve divided by three" like this:
>
> $$\frac{12}{3}$$

So,

$$3 \times n = 12$$
$$n = \frac{12}{3}$$
$$n = 4$$

Review the meaning of $\frac{12}{3}$. The bar used between the top numeral and the bottom numeral shows that one number is divided by the other; that the top numeral names the number of the set to be partitioned and the bottom numeral shows either the number of members in each subset or the number of subsets into which the set is partitioned.

Ask if we can solve the equation $n \times 3 = 12$ in the same way. (Yes.)

$$n \times 3 = 12$$

$$n = \frac{12}{3}$$

$$n = 4.$$

Emphasize the fact that when we divide one number by another, the result is called the quotient. In this case, 4 is the quotient. The number by which we divide is the divisor. (Above, 3 is the divisor.)

Follow the same procedure using some of the equations listed below.

$n \times 7 = 28$　　$5 \times n = 30$

$4 \times n = 32$　　$7 \times n = 28$

$3 \times n = 18$　　$n \times 5 = 25$

$n \times 5 = 20$　　$n \times 3 = 27$

Pupil's book, page 441: provides practice in solving multiplication equations by division.

page 442: provides practice in writing division expressions in two ways.

Solving Equations

The first one has been done for you.

$n \times 9 = 36$ $n = \frac{36}{9}$ $n = 4$	$7 \times n = 56$ $n = \frac{56}{7}$ $n = 8$
$5 \times n = 45$ $n = \frac{45}{5}$ $n = 9$	$n \times 6 = 54$ $n = \frac{54}{6}$ $n = 9$
$6 \times n = 42$ $n = \frac{42}{6}$ $n = 7$	$n \times 4 = 16$ $n = \frac{16}{4}$ $n = 4$
$7 \times n = 49$ $n = \frac{49}{7}$ $n = 7$	$n \times 5 = 35$ $n = \frac{35}{5}$ $n = 7$
$4 \times n = 28$ $n = \frac{28}{4}$ $n = 7$	$8 \times n = 24$ $n = \frac{24}{8}$ $n = 3$

Two Ways of Writing a Division Example

The first example in each column has been done for you.

$35 \div 7 = \frac{35}{7} = 5$

$42 \div 6 = \frac{42}{6} = 7$

$81 \div 9 = \frac{81}{9} = 9$

$45 \div 5 = \frac{45}{5} = 9$

$72 \div 8 = \frac{72}{8} = 9$

$40 \div 5 = \frac{40}{5} = 8$

$63 \div 9 = \frac{63}{9} = 7$

$24 \div 6 = \frac{24}{6} = 4$

$56 \div 7 = \frac{56}{7} = 8$

$\frac{18}{2} = 18 \div 2 = 9$

$\frac{27}{3} = 27 \div 3 = 9$

$\frac{32}{4} = 32 \div 4 = 8$

$\frac{32}{8} = 32 \div 8 = 4$

$\frac{30}{5} = 30 \div 5 = 6$

$\frac{49}{7} = 49 \div 7 = 7$

$\frac{28}{4} = 28 \div 4 = 7$

$\frac{18}{3} = 18 \div 3 = 6$

$\frac{36}{4} = 36 \div 4 = 9$

IX-2. Finding quotients

Objective: To help children find the quotient when the quotient is not a whole number.

Vocabulary: (No new words.)

Materials: Number line, regions of various kinds.

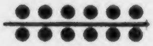
Suggested Procedure:

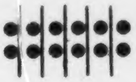
Begin with a problem situation.

> Bill had 12 cookies. He wanted to share them with his friend. How many cookies did each boy receive?

Tell the children that this is a very easy problem but you want them to solve it in several ways. Ask them for suggestions and accept the ways they offer. Possibilities are:

1. Make an array. It may be partitioned in two different ways.





2. Solve the equation: $2 \times n = 12$ or $n \times 2 = 12$

$$n = \frac{12}{2} \qquad n = \frac{12}{2}$$

$$n = 6 \qquad n = 6$$

3. Just knowing your multiplication and division facts:

   $2 \times \underline{6} = 12$ or $12 \div 2 = 6$

Continue the discussion.

> Suppose Bill told his friend that he would give him one-half of the cookies. How do you think this would be solved? (Possible

answers -- we did this when we made the array. Split the set of 12 into 2 equivalent subsets.)

Ask a child to write the fraction on the chalkboard. Review the meaning of the 1 (one set) and the 2 (split into 2 equivalent subsets).

Ask a child, "What is $\frac{1}{2}$ of 12?"

Write on the chalkboard: $\frac{1}{2}$ of $12 = \frac{12}{2} = 6$.

Follow with several examples taken from those below. If necessary, partition the sets into the number of equivalent subsets designated by the denominator of the fraction.

$\frac{1}{2}$ of 6

$\frac{1}{2}$ of 18

$\frac{1}{2}$ of 8

$\frac{1}{3}$ of 12

$\frac{1}{3}$ of 15

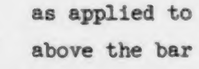
$\frac{1}{3}$ of 21

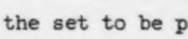
$\frac{1}{4}$ of 8

$\frac{1}{4}$ of 12

$\frac{1}{4}$ of 40

Ask the children how many cookies Bill would give his friend if he had 13 cookies. Hopefully, some child will suggest that each boy would get 6 cookies and $\frac{1}{2}$ of a cookie. If no one does so, make an array to show that there would be 1 cookie remaining after the array had been formed.





Ask the children what could be done with the remaining cookie. (Cut it in half.) Review the meaning of $\frac{1}{2}$ as applied to $\frac{1}{2}$ of 1. In this case, the numeral above the bar names the number of the set to be partitioned and the numeral below the bar names the number of congruent parts into which the set has been divided.

Write on the chalkboard: $\frac{1}{2}$ of $13 = \frac{13}{2} = 6 + \frac{1}{2}$

Does $6 + \frac{1}{2}$ tell how many cookies each boy received? (Yes.)

Explain that $6 + \frac{1}{2}$ is often written $6\frac{1}{2}$, without a plus sign, and is read "six and one half".

Consider other examples, such as:

$\frac{1}{2}$ of 15 $\frac{1}{3}$ of 10 $\frac{1}{4}$ of 9 $\frac{1}{5}$ of 11

$\frac{1}{2}$ of 9 $\frac{1}{3}$ of 7 $\frac{1}{4}$ of 25 $\frac{1}{5}$ of 21

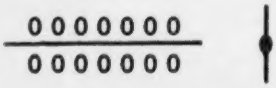
Pupil's book, page 443: requires the child to find quotients that are not whole numbers.

page 444: provides review in shading fractional parts.

Fair Shares

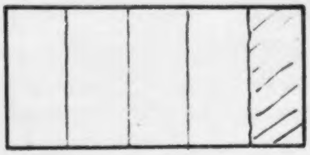
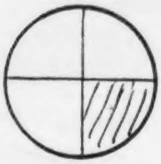
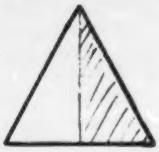
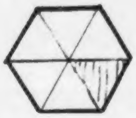
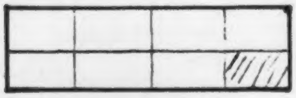
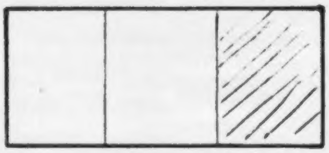
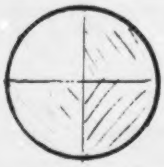
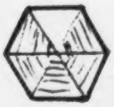
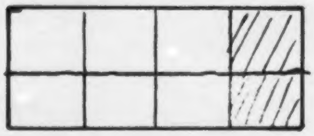
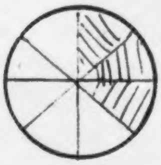
Show your partition. Work the example.

The first one has been done for you.

$\frac{1}{2}$ of 15 = $\frac{15}{2}$ = 7 + $\frac{1}{2}$ = $7\frac{1}{2}$	
$\frac{1}{3}$ of 16 = $5 + \frac{1}{3}$ = $5\frac{1}{3}$	
$\frac{1}{4}$ of 13 = $3 + \frac{1}{4}$ = $3\frac{1}{4}$	
$\frac{1}{5}$ of 21 = $4\frac{1}{5}$	
$\frac{1}{6}$ of 19 = $3\frac{1}{6}$	
$\frac{1}{7}$ of 22 = $3\frac{1}{7}$	
$\frac{1}{8}$ of 25 = $3\frac{1}{8}$	
$\frac{1}{9}$ of 28 = $3\frac{1}{9}$	

Fractions

Divide the region into congruent parts. Shade the part that represents the fraction.

	$\frac{1}{5}$		$\frac{1}{4}$
	$\frac{1}{2}$		$\frac{1}{6}$
	$\frac{1}{8}$		$\frac{1}{3}$
	$\frac{3}{4}$		$\frac{5}{6}$
	$\frac{2}{8}$		$\frac{3}{8}$

IX-3. Finding the number that names part of a set

Objective: To develop a technique for finding the number that names part of a set when the set has more than one member.

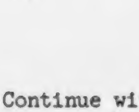
Vocabulary: (No new words.)

Materials: Arrays.

Suggested Procedure:

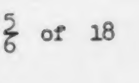
> Suppose Bill's mother said that the boys could have two-thirds of 12 cookies. How many cookies would Bill take?

Ask the children how they would find $\frac{2}{3}$ of 12. Possibly they will suggest finding $\frac{1}{3}$ of 12 and then multiplying the result by 2. If not, use the array to find $\frac{1}{3}$ of 12. This number is to be multiplied by 2 since if $\frac{1}{3}$ of 12 = 4, then $\frac{2}{3}$ of 12 would be twice as much or 2 × 4.



$\frac{1}{3}$ of 12

$\frac{2}{3}$ of 12 = ________



$\frac{2}{3}$ of 12

$\frac{1}{3}$ of 12 = $\frac{12}{3}$ = 4

2 × 4 = 8

$\frac{2}{3}$ of 12 = 8

Continue with similar examples.

$\frac{5}{6}$ of 18

$\frac{2}{5}$ of 10

$\frac{5}{8}$ of 24

$\frac{3}{4}$ of 28

$\frac{3}{5}$ of 30

$\frac{2}{9}$ of 18

$\frac{3}{5}$ of 35

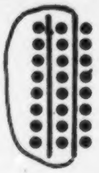
$\frac{4}{9}$ of 27

$\frac{7}{8}$ of 48.

Pupil's book pages 445-446: provide practice in finding the number that names part of a set.

Finding the Number That Names Part of a Set

Work each example. Make an array to show that your answer is correct. The first one has been done for you.

1) $\frac{2}{3}$ of 24  $\frac{1}{3}$ of 24 = $\frac{24}{3}$ = 8 2 × 8 = 16 $\frac{2}{3}$ of 24 = 16	2) $\frac{3}{5}$ of 25 $\frac{3}{5}$ of 25 = 15
3) $\frac{4}{9}$ of 36 $\frac{4}{9}$ of 36 = 16	4) $\frac{5}{7}$ of 21 $\frac{5}{7}$ of 21 = 15
5) $\frac{3}{4}$ of 16 $\frac{3}{4}$ of 16 = 12	6) $\frac{5}{8}$ of 24 $\frac{5}{8}$ of 24 = 15

7) $\frac{2}{3}$ of 27

$\frac{2}{3}$ of $27 = 18$.

8) $\frac{5}{7}$ of 35

$\frac{5}{7}$ of $35 = 25$

9) $\frac{5}{6}$ of 18

$\frac{5}{6}$ of $18 = 15$

10) $\frac{3}{8}$ of 16

$\frac{3}{8}$ of $16 = 6$

11) Use the symbols $>$, $<$ or $=$ to make these true statements.

$\frac{3}{4}$ of 12 $>$ $\frac{2}{3}$ of 12

$\frac{2}{3}$ of 15 $=$ $\frac{5}{6}$ of 12

$\frac{5}{8}$ of 16 $>$ $\frac{4}{9}$ of 18

$\frac{8}{9}$ of 27 $>$ $\frac{5}{6}$ of 24

$\frac{2}{9}$ of 45 $<$ $\frac{2}{5}$ of 30

$\frac{3}{8}$ of 48 $<$ $\frac{4}{7}$ of 42

$\frac{5}{9}$ of 63 $<$ $\frac{6}{7}$ of 49

$\frac{3}{7}$ of 14 $=$ $\frac{2}{7}$ of 21

The number that names part of a set

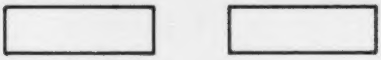
Five girls were playing together. Mary's mother gave them 2 candy bars. They didn't know how to cut up the bars. Let's see if we can help them. How many children are to share the candy? (5.)

How many candy bars do they have? (2.)

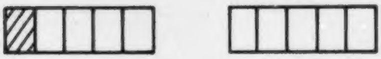
Would each child receive $\frac{1}{5}$ of 2 candy bars? (Yes.)

Write on the chalkboard: $\frac{1}{5}$ of $2 = \frac{2}{5}$.

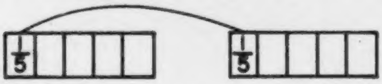
Draw a sketch of the candy bars on the chalkboard.



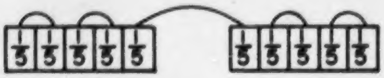
Ask the children how the bars should be cut if each child is to receive $\frac{1}{5}$ of 2 candy bars. Probably some child will suggest cutting each bar into 5 pieces. If not, suggest this.



Shade 1 of the 5 congruent parts of 1 bar. Ask the children what part of the bar is shaded. ($\frac{1}{5}$.) Follow with the question, "Will each child receive just a piece this size?" (No.) Bring out the fact that this would be $\frac{1}{5}$ of 1 or $\frac{1}{5}$ but the children are supposed to receive two-fifths of the candy. Suggest that each child be given $\frac{1}{5}$ of each bar. Ask if each child would then have $\frac{2}{5}$ of the candy. (Yes.) Demonstrate that this is true by making this drawing on the chalkboard.



Then suggest it might be easier to just pass out the candy by giving $\frac{2}{5}$ at one time.



Discuss the fact that the first drawing shows that $\frac{1}{5}$ of $2 = \frac{2}{5}$; $\frac{1}{5}$ from each bar. The second drawing shows that $\frac{2}{5}$ of $1 = \frac{2}{5}$.

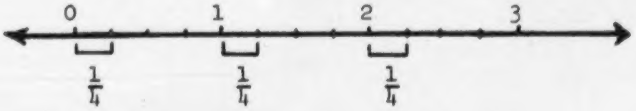
Since $\frac{1}{5}$ of $2 = \frac{2}{5}$ and $\frac{2}{5}$ of $1 = \frac{2}{5}$, then $\frac{1}{5}$ of $2 = \frac{2}{5}$ of 1.

Use another problem to demonstrate the same thing on the number line.

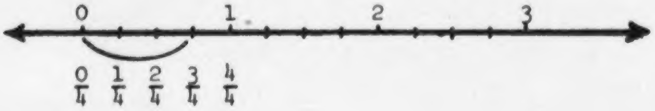
> Four boys were given 3 bars of candy.
> How much candy will each boy receive?

Ask the children if they know how much candy each boy will get. Possibly some one will suggest $\frac{1}{4}$ of 3 bars and/or $\frac{3}{4}$ of 1 or $\frac{3}{4}$ bar. Try to get both of these answers.

Show on the number line that both of these answers are correct.



If each child received $\frac{1}{4}$ of each of 3 bar, this is equal to $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ of one bar as demonstrated below:


0
1
2
3
$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

$\frac{1}{4}$ of $3 = \frac{3}{4}$ and $\frac{3}{4}$ of $1 = \frac{3}{4}$ so $\frac{1}{4}$ of $3 = \frac{3}{4}$ of 1.

Discuss with the children which solution seems easier. Of course, they may take their own preference although for most of them the second drawing is easier.

Use a few other examples.

5 boys - 4 bars
7 boys - 6 bars
6 boys - 5 bars

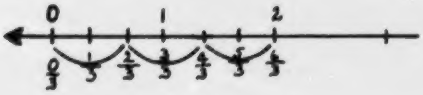
Pupil's book, page 447: reinforces this idea.

Using the Number Line

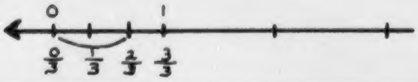
Use the number line to show that your answer is correct.

The first one has been done for you.

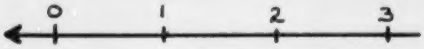
$\frac{1}{3}$ of 2 = $\frac{2}{3}$


0
1
2
$\frac{0}{3}$
$\frac{1}{3}$
$\frac{2}{3}$
$\frac{3}{3}$
$\frac{4}{3}$
$\frac{5}{3}$
$\frac{6}{3}$

$\frac{2}{3}$ of 1 = $\frac{2}{3}$


0
1
$\frac{0}{3}$
$\frac{1}{3}$
$\frac{2}{3}$
$\frac{3}{3}$

$\frac{1}{5}$ of 3 = $\frac{3}{5}$



$\frac{3}{5}$ of 1 = $\frac{3}{5}$

$\frac{1}{7}$ of 2 = $\frac{2}{7}$

$\frac{2}{7}$ of 1 = $\frac{2}{7}$

$\frac{1}{8}$ of 3 = $\frac{3}{8}$

$\frac{3}{8}$ of 1 = $\frac{3}{8}$

$\frac{1}{6}$ of 2 = $\frac{2}{6}$ (or $\frac{1}{3}$)

$\frac{2}{6}$ of 1 = $\frac{2}{6}$ (or $\frac{1}{3}$)

IX-4. A technique for finding quotients

Objective: To develop the long division algorithm to find quotients that are not whole numbers.

Vocabulary: (No new words.)

Materials: Manipulative materials.

Suggested Procedure:

Ask how it is possible to divide 45 by 9 without using set materials. (Some one should know that $9 \times 5 = 45$ or that $45 \div 9 = 5$.)

> Sometimes we don't know a multiplication fact so we learned how to use different facts to help us find the quotient. We are going to review this procedure and the way we recorded our thinking.

$$\begin{array}{r|rl} 9\overline{)\,45} & & \\ \underline{27} & \underline{3} & (3 \times 9 = 27)\ \text{Think of making 3 sets of 9.} \\ 18 & & \\ \underline{18} & \underline{2} & (2 \times 9 = 18)\ \text{Think of making 2 more sets of 9.} \\ & 5 & \end{array}$$

$45 \div 9 = 5$

Suppose we used the fact $4 \times 9 = 36$

$$\begin{array}{r|rl} 9\overline{)\,45} & & \\ \underline{36} & \underline{4} & \text{Think of making 4 sets of 9.} \\ 9 & & \\ \underline{9} & \underline{1} & \text{Think of making 1 more set of 9.} \\ & & \end{array}$$

Again the equation $45 \div 9 = 5$ or $\frac{45}{9} = 5$.

Ask if there is just one fact that can be used only once.

$$\begin{array}{r|rl} 9\overline{)\,45} & & \\ \underline{45} & \underline{5} & (5 \times 9 = 45) \end{array}$$

If you feel that your class needs practice, continue with other examples and explore ways of finding any quotient should you not be able to think of the fact.

Then consider a problem situation.

> Mr. Smith brought 25 candy bars home to his 4 children. How much of the candy will each child receive?

Use the long division algorithm and begin with any multiple of 4 which the children suggest. A possibility is:

$$\begin{array}{rr|l} 4\,) & 25 & \\ & 12 & 3 \\ \cline{2-3} & 13 & \\ & 12 & 3 \\ \cline{2-3} & 1 & 6 \end{array}$$

Ask if there is one fact that can be used only once.

$$\begin{array}{rr|l} 4\,) & 25 & \\ & 24 & 6 \\ \cline{2-3} & 1 & \end{array}$$

There are 6 sets of 4 in 25 with 1 remaining. Remind the children that we can think of 25 as 24 + 1. Ask if the 24 has been divided by 4. (Yes.) Then ask if the 1 has been divided by 4. (No.) Suggest that the children would each receive 6 bars of candy and probably would throw the other one away. The children will certainly object to this and probably will suggest that the bar be divided into 4 parts. If they do not, then you offer the idea. Ask how much of the bar each child will receive. ($\frac{1}{4}$.) Follow with the question, "How many bars does each child get?" (6 bars and $\frac{1}{4}$ of a bar or $6 + \frac{1}{4}$.)

Return to the notation on the chalkboard. Write the complete quotient as shown.

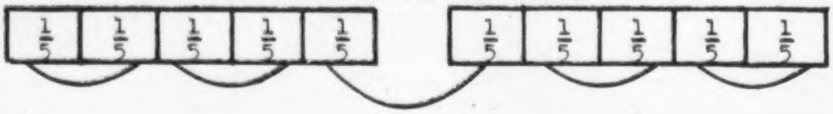
$$\begin{array}{rr|l} & 6 + \frac{1}{4} & \\ \cline{2-3} 4\,) & 25 & \\ & 24 & 6 \\ \cline{2-3} & 1 & 6 \end{array}$$

Use another example. A possibility is given below.

$$5\,\overline{)\begin{array}{r|l} 42 & \\ \underline{20} & \underline{4} \\ 22 & \\ \underline{20} & \underline{4} \\ 2 & 8 \end{array}} \qquad 5\,\overline{)\begin{array}{r|l} 42 & \\ \underline{40} & \underline{8} \\ 2 & 8 \end{array}}$$

Ask the children what the quotient is. If anyone suggests 8, point out that $40 \div 5 = 8$ but the example is $42 \div 5$. The 40 has been divided by 8 but the 2 still remains to be divided by 5. Ask the children how we write 2 divided by 5. ($\frac{2}{5}$.) The quotient is $8 + \frac{2}{5}$. If there is any confusion, clarify by asking the children how many whole candy bars each child would receive if there were 42 candy bars and 5 children. (8.) Then 2 candy bars remain to be divided among the 5 children. Each would receive $\frac{1}{5}$ of 2.

Make this illustration on the chalkboard.



Each bar has been divided into 5 congruent parts. Therefore, each part represents $\frac{1}{5}$ of a bar. Each child will receive 2 parts or $\frac{2}{5}$ of 1 bar. The total amount of candy received by each child will be $8 + \frac{2}{5}$.

Pupil's book, pages 448-450; provides practice in finding quotients that are not whole numbers.

Finding Quotients

The first example has been done for you.

$9 + \frac{1}{3}$ 3) 28 27 9 1 9	$8 + \frac{1}{2}$ 2) 17
$5 + \frac{1}{5}$ 5) 26	$5 + \frac{1}{4}$ 4) 21
$6 + \frac{1}{3}$ 3) 19	$6 + \frac{1}{5}$ 5) 31
$7 + \frac{1}{4}$ 4) 29	$5 + \frac{1}{3}$ 3) 16

Finding Quotients

The first example has been done for you.

22 ÷ 5 = $4 + \frac{2}{5}$ $\begin{array}{r\|l} 5)\,22 & \\ 20 & 4 \\ \hline 2 & 4 \end{array}$	32 ÷ 6 = $5 + \frac{2}{6}$
30 ÷ 7 = $4 + \frac{2}{7}$	37 ÷ 5 = $7 + \frac{2}{5}$
45 ÷ 7 = $6 + \frac{3}{7}$	38 ÷ 9 = $4 + \frac{2}{9}$
29 ÷ 5 = $5 + \frac{4}{5}$	43 ÷ 8 = $5 + \frac{3}{8}$

Finding Quotients

The first example has been worked for you.

$\frac{27}{5} = 5 + \frac{2}{5}$ $5\overline{)27}$ 25 5 2 5	$\frac{52}{7} = 7 + \frac{3}{7}$
$\frac{75}{8} = 9 + \frac{3}{8}$	$\frac{59}{6} = 9 + \frac{5}{6}$
$\frac{35}{4} = 8 + \frac{3}{4}$	$\frac{89}{9} = 9 + \frac{8}{9}$
$\frac{66}{8} = 8 + \frac{2}{8}$	$\frac{45}{8} = 5 + \frac{5}{8}$

IX-5. Solving problems

Objective: To develop skill in solving story problems using techniques of division.
To increase understanding that because of the relationship between multiplication and division different equations may be used to help solve story problems.

Vocabulary: (No new words.)

Materials: Flannel board or magnetic board with materials to show sets of ten and ones.

Suggested Procedure:

Read the following story problem to the children:

> Mrs. March asked John to pass the counting blocks to the 8 children in his group. The box of blocks was full and on the box John saw "96 Blocks". John passed out all of the blocks. If each child received the same number of blocks how many blocks would each child get?

Ask a child to restate for the class what is to be found.

> (How many blocks would each child get if John passes out all of the blocks and gives the same number to each child?)
>
> What information is given to help us solve the problem? (There are 96 blocks for 8 children.)
>
> Would it help to think about the blocks as a set with 96 members? What operation could we use? (We could partition the set of blocks into equivalent subsets.)

Display 9 sets of ten and 6 ones on the flannel board.

Is this set equivalent to the set of blocks? (Yes.)

Encourage the children to discuss how they could partition the set on the flannel board into equivalent subsets. Ask a child to write the equation which is related to the problem.

$(96 \div 8 = n)$

Through pupil participation bring out that each subset would have one set of ten and the remaining set of ten and the six ones would need to be separated evenly among the 8 subsets. This would involve exchanging one set of ten and the 6 ones to 16 ones. Each subset would then have 12 members. Have the equation completed and the solution to the problem stated.

(Each child would get 12 blocks.)

Read the following story:

> Mrs. March asked Molly to get the colored chalk ready to use. Molly was to put the same number of pieces in each of 6 small boxes. There were 75 pieces in the box of chalk that Mrs. March gave to Molly. How many pieces of chalk will Molly put in each of the small boxes?

Ask a child to write the equation which is related to the problem.

$(75 \div 6 = n)$

Using the same procedure used for the first problem show a set equivalent to the set of chalk. (75 pieces.)
Through pupil participation determine that each subset will have 12 members and there are 3 members not used.
Suggest to the children that they think about what Molly might do if she had 3 pieces of chalk left.

> Perhaps Molly knows that these are new pieces of chalk and there has been no chance for them to get broken. She also knows that sometimes Mrs. March suggests that it is easier to use the chalk when it is in shorter pieces. What might Molly do?

(She might break each of the remaining pieces in half and put $\frac{1}{2}$ piece of chalk in each box.)

Ask a child to complete the equation.

$(75 \div 6 = 12\frac{1}{2})$

The solution to the problem should be stated. (Molly put $12\frac{1}{2}$ pieces of chalk in each of the small boxes.)

Ask the children to think with you about a different ending to the story.

> Molly has separated the chalk among the six boxes. She has three pieces left. Molly remembers that Mrs. March has often asked the children to be careful not to break the chalk. Mrs. March says that longer pieces are easier to hold in your hand. Would Molly break the chalk then? (No.)
>
> Could we still say, "Molly put $12\frac{1}{2}$ pieces of chalk in each of the small boxes," is the solution to our problem?
>
> (No, we will have to say, "Molly put 12 pieces in each box and had 3 pieces of chalk left".)

Draw the children's attention to the equation $75 \div 6 = 12\frac{1}{2}$ and ask if this will have to be changed. Help the children understand that the equation is correct as it is. The equation tells only about number, not about chalk. When the "=" symbol is used the number on the left must be "exactly equal" to the number on the right.

This is a wonderful opportunity to show that the equation is not the solution to the story problem. The equation helps us find the number used to solve the story problem. The solution to the equation may not be a practical solution to the story problem. In this case we may choose a number near the solution of the equation.

Read the following story:

> Jim and his 2 brothers were each building a bird house. They needed screws so Father went to the store and bought 4 dozen screws. How many screws will each boy get?

Encourage the children to solve the problem using a procedure similar to that used for the first two problems. Determine how many screws were in the set of screws. Talk about the number of subsets and the number of members in each subset. Have the equation written and the solution to the problem given.

(48 ÷ 3 = n, 16 = n, Each boy will get 16 screws.)

Continue by reading more about Jim and his brothers:

> Jim and his 2 brothers wanted to put perches on the houses for the birds to sit on. Father bought a package of ten pegs for the boys to use. The boys wanted to put the same number of perches on each birdhouse. How many perches could they put on the houses if each peg will make one perch?

After discussing the story, ask a child to write the equation and complete the computation on the board.

(10 ÷ 3 = n, $3\frac{1}{3}$ = n)
Will the boys have $3\frac{1}{3}$ perches on their birdhouses? (No, the solution to the story problem is that the boys will each have three perches and there will be one peg left over.)

Pupil's book, page 451: Go over the directions with the children, read the story problems if necessary, and direct the children to complete the page independently.

Note: You should be constantly aware of the fact that for many story problems there are several ways of solving them and that one way may be just as correct as another way. As long as the child can explain how he reached the conclusion he should not be forced to use a way that may seem more logical to the teacher or to another child.

The child should be encouraged to relate the equation to the problem but he may then show the same relationship between the numbers using a different equation.

Write the following equations on the board:

27 = 3 x n   27 ÷ 3 = n

Read this story to the children.

> 27 children were playing on the playground.
> They decided to split into 3 small groups.
> How many children were in each of the groups?

Ask the children to indicate by raising their hands which equation fits the problem. It is hoped that the children will provide the information that either equation is correct. Elicit from the children the explanation for each equation.

For example, $27 = 3 \times n$ You know that there are 27 children all together and there are 3 groups. 27 children is the number of children in each group, 3 times. You can use the equation $27 = 3 \times n$ to help solve the problem. $27 \div 3 = n$ There are 27 children all together and there are 3 groups. If you separate the 27 children into 3 groups you will know how many children in each group. You can use the equation $27 \div 3 = n$ to help solve the problem.

Complete the equations and have the solution to the problem stated.

(There were 9 children in each group.)

Using the following story problems encourage the children to show different equations and explain how they could be used to help solve the problem.

1. Jim's team won the game 24 to 12. If each touchdown counts 6 points how many touchdowns did Jim's team make?

   $24 = 6 \times n$ $\qquad$ $24 \div 6 = n$

   (Jim's team made 4 touchdowns.)

2. Father drove at the same speed for 3 hours and traveled 150 miles. How far did he drive each hour?

150 = 3 x n   150 ÷ 3 = n

(Father drove 50 miles each hour.)

3. Jane had 9 pieces of candy for each of 3 girls. How many pieces of candy will each girl get?

   9 = 3 x n   9 ÷ 3 = n

   (Each girl got 3 pieces of candy.)

4. Mary saw 15 flowers in Mother's plant box. 4 plants had no flowers and 3 plants each had the same number of flowers. If there were 15 flowers on the 3 plants how many flowers were on each plant?

   15 = 3 x n   15 ÷ 3 = n

   (Each of the 3 plants had 5 flowers on it.)

5. John had 20 rocks in his rock collection. He put them in a box in rows. Each of the 5 rows had the same number of rocks. How many rocks were in each row?

   20 = 5 x n   20 ÷ 5 = n

   (There were 4 rocks in each row.)

6. Mother put some bacon on each of 6 plates. She put an equal number of pancakes on each of the plates. All together she put 18 pancakes on the plates. How many pancakes were on each plate?

   18 = 6 x n   18 ÷ 6 = n

   (Each plate had 6 pancakes on it.)

Solving Problems

Write the equation that will help you solve the problem. Equations may vary

Write the sentence that tells the solution to the problem.

1. The 21 children on the playground decided to play a team game. It took 6 children for each team. How many teams could they make with the 21 children?

$n = \frac{21}{6}$ $n = 3\frac{3}{6}$ They could make 3 teams and they had 3 extra children.

2. Judy and 3 of her friends were playing house in the back yard. Judy's mother brought a plate of cookies for the girls to eat. Judy wanted each girl to have the same number of cookies. There were 10 cookies on the plate. How many cookies did each girl get?

$n = \frac{10}{4}$ $n = 2\frac{2}{4}$ Judy gave each girl $2\frac{2}{4}$ or $2\frac{1}{2}$ cookies.

3. Mrs. White had 23 tulip bulbs to plant. She planted the same number of bulbs in each of three rows. How many bulbs did she plant in each row?

$n = \frac{23}{3}$ $n = 7\frac{2}{3}$ Mrs. White planted 7 bulbs in each row. She had 2 bulbs left over.

The following is a list of all those who participated in the preparation of this volume:

Leslie Beatty, Chula Vista City School District, Chula Vista, California
Truman Botts, University of Virginia
Leon W. Cohen, University of Maryland
Zigmund Drapalski, WTVS, Detroit Public Schools, Detroit, Michigan
Jean Dunn, Palo Alto Unified School District, Palo Alto, California
Wade Ellis, Oberlin College, Oberlin, Ohio
Mary O. Folsom, University of Miami, Florida
Mary E. Giamperoli, Edith C. Baker School, Chestnut Hill, Massachusetts
Leonard Gillman, University of Rochester, Rochester, New York
E. Glenadine Gibb, State College of Iowa
Muriel Greig, McColl School, Detroit, Michigan
Adrien L. Hess, Montana State College
Stanley B. Jackson, University of Maryland
John L. Kelley, University of California, Berkeley
Sharon Logan, Oak Ridge Elementary School, Arlington, Virginia
William F. McClintock, Stanislaus State College, Turlock, California
Mary McCulloch, University School, Northern Illinois University, DeKalb, Illinois
Patricia Michels, Joaquin Miller School, Oakland, California
Rose Mijanovich, Joaquin Miller School, Oakland, California
Mildred Pierce, Humbert School, Cedar Falls, Iowa
Frank W. Sinden, Bell Telephone Laboratories, Murray Hill, New Jersey
Jane Stenzel, Cambrian Elementary School District, San Jose, California
J. Fred Weaver, Boston University